

M-theory on seven-dimensional manifolds with $SU(3)$ structure

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ABSTRACT: In this paper we study M-theory compactifications on seven-dimensional manifolds with $SU(3)$ structure. As such manifolds naturally pick out a specific direction, under certain assumptions regarding the expansion basis the resulting effective theory can be cast into a form which is similar to type IIA compactifications to four dimensions. We derive the gravitino mass matrix in four dimensions and show that for different internal manifolds (torsion classes) the vacuum preserves either no supersymmetry, or $\mathcal{N} = 2$ supersymmetry or, through spontaneous partial supersymmetry breaking, $\mathcal{N} = 1$ supersymmetry. For the latter case we derive the effective $\mathcal{N} = 1$ theory and give explicit examples where all the moduli are stabilised without the need of non-perturbative effects.

KEYWORDS: Superstring Vacua, M-Theory, Flux compactifications.

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1. Introduction

The low energy limit of M-theory, that is eleven-dimensional supergravity, forms arguably the most natural starting point from which we hope to recover observable physics from a fully consistent theory. The first issue to address is of course the fact that we observe four dimensions and the most phenomenologically successful approach so far has been to single out one of the space dimensions as independent of the other nine. Compactifying on this dimension then leads to type IIA string theory [1–3] which can then be compactified to four dimensions on a six-dimensional Calabi-Yau. The dimension may also be taken to be an interval, and then compactifying on a Calabi-Yau leads to a Brane-world scenario [4]. If we do not require the existence of such a special trivially fibred direction we should consider compactifying on seven dimensional manifolds. The possible contenders for such manifolds are required by supersymmetry to have special holonomy and until recently the main body of work has concentrated on manifolds with G_2 -holonomy that lead to Minkowski space in four dimensions and preserve $\mathcal{N} = 1$ supersymmetry [5]. These compactifications lead to massless scalar fields in four dimensions that are known as moduli and an important first phenomenological step is to lift these flat directions. In string theory flux compactifications have proved very successful in achieving this (for a review see [6]) and in M-theory there has been some success in the case of G_2 -manifolds [7–9]. A feature of flux compactifications is that flux on the internal manifold will back-react on the geometry and in general induce torsion and warping on the manifold deforming its special holonomy to the more general property of a G structure [10, 11]. To take this back-reaction into account we should therefore consider compactifications on manifolds with a particular G structure. Compactifications that derive the four dimensional theory have been done for the case of manifolds with G_2 structure [9, 12–14]. Eleven dimensional solutions that explore the structure of the vacuum have been studied for the cases of $SU(2)$, $SU(3)$ and G_2 structure in [15–22]. An interesting point to come out of these studies is that compactifications on manifolds with $SU(3)$ structure have a much richer vacuum spectrum than manifolds with G_2 structure. Indeed there are solutions that preserve only $\mathcal{N} = 1$ supersymmetry in the vacuum putting them on an equal phenomenological grounding with G_2 compactifications in that respect. There are however many phenomenologically appealing features that are not present in the G_2 compactifications such as warped anti-deSitter solutions and solutions with non-vanishing internal flux.

In this paper we will study compactifications on manifolds with $SU(3)$ structure. We will see that because the $SU(3)$ structure naturally picks out a vector on the internal manifold these compactifications can be cast into a form that is similar to type IIA compactifications on $SU(3)$ structure manifolds [23]. However unlike in (massless) type IIA, we will show that, under certain assumptions regarding the expansion basis discussed in section 3.2.1, it is possible to find purely perturbative vacua with all the moduli stabilised that preserve either $\mathcal{N} = 2$ or $\mathcal{N} = 1$ supersymmetry [24–26]. Moreover, as also remarked in [27], such compactifications offer the possibility to obtain charged scalar fields which reside in the $\mathcal{N} = 2$ vector multiplets rather than in the hypermultiplets as realised so far in most cases (see for example [6]).

We will begin this paper with a discussion of the notion of G structures and the idea of mass hierarchies between various G structures. In section 3 we will perform a reduction of eleven-dimensional supergravity on a general manifold with $SU(3)$ structure deriving the kinetic terms for the four-dimensional scalar fields and the four-dimensional gravitini mass matrix. The mass matrix will then be used to explore the amount of supersymmetry preserved by various manifolds. We will begin by looking at vacua that preserve $\mathcal{N} = 2$ supersymmetry in section 4. We will first derive the most general $\mathcal{N} = 2$ solution and use it as a check on the mass matrix. We will then show how this solution can be used to find explicit vacua of an example manifold. In section 5 we will move on to the more phenomenologically interesting $\mathcal{N} = 1$ vacua and will show that some manifolds will induce spontaneous partial supersymmetry breaking that will lead to an $\mathcal{N} = 1$ effective theory. We will derive this theory and go through an explicit example of moduli stabilisation. This will also serve as an interesting example of a mass gap between G structures. Finally, in the Appendices, we present our conventions and some technical details related to the calculations we perform in the main text.

Note added: while this manuscript was prepared for publication another paper appeared, [57], which has some overlap with the issues discussed in this paper. Further to this we were informed of work in progress which also relates to the discussed issues [58].

2. G structures

In this section we briefly discuss the notion of a G structure and the two particular cases of G_2 - and $SU(3)$ structure in seven-dimensions. For a more thorough introduction to G structures we refer the reader to [10, 11]. A manifold is said to have G structure if the structure group of the frame bundle reduces to the group G. In practice this translates into the existence of a set of G-invariant forms and spinors on such manifolds.

In general these forms are not covariantly constant with respect to the Levi-Civita connection, which would imply that the holonomy group of the manifold is reduced to G. The failure of the Levi-Civita connection to have reduced holonomy G is measured by the *intrinsic torsion*. In turn, the intrinsic torsion, and in particular its decomposition in G-representations, is used to classify such manifolds with G structure. In the following we will give a couple of examples of G structures defined on seven-dimensional manifolds which we will use in this paper.

2.1 G_2 structure in seven dimensions

A seven-dimensional manifold with G_2 structure has a globally defined G_2 -invariant, real and nowhere-vanishing three-form φ which can be defined by a map to an explicit form in an orthonormal basis [28]. Alternatively, manifolds with G_2 structure feature a globally defined, G_2 -invariant, Majorana spinor ϵ . Note that we shall work in a basis where Majorana spinors are real. In terms of this spinor the G_2 form, φ is defined as

$$\varphi_{mnp} = i\epsilon^T \gamma_{mnp} \epsilon, \tag{2.1}$$

with the spinor normalisation $\epsilon^T \epsilon = 1$.

Using the G_2 structure form φ we can write

$$\begin{aligned} d\varphi &= W_1 \star \varphi - \varphi \wedge W_2 + W_3, \\ d(\star\varphi) &= \frac{4}{3} \star \varphi \wedge W_2 + W_4, \end{aligned} \tag{2.2}$$

where W_1, \dots, W_4 are the four torsion classes. In terms of G_2 representations W_1 is a singlet, W_2 a vector, W_3 a **27** while W_4 transforms under the adjoint representation, **14**. For further reference we note here that manifolds with only $W_1 \neq 0$ are called weak- G_2 manifolds and they are the most general solutions of the Freund-Rubin Ansatz [29, 30].

2.2 $SU(3)$ structure in seven dimensions

Manifolds with $SU(3)$ structure are more familiar in the context of six dimensions. In particular, the most important representatives are the Calabi–Yau manifolds for which the intrinsic torsion vanishes identically (ie, as explained before they have $SU(3)$ holonomy). On the other hand, seven-dimensional manifolds with $SU(3)$ structure were less studied partly due to the fact that for the case of no torsion where the holonomy group of the manifold is $SU(3)$ the seven-dimensional manifold is just a direct product of a Calabi–Yau manifold and a circle. Therefore studying M-theory on such manifolds is equivalent to studying type IIA string theory on a Calabi-Yau. Once some torsion classes are non-vanishing a non-trivial fibration is generated thereby making such studies different to type IIA compactifications.

An $SU(3)$ structure on a seven dimensional manifold implies the existence of two globally defined, nowhere-vanishing Majorana spinors ϵ_1 and ϵ_2 which are independent in that they satisfy $\epsilon_1^T \epsilon_2 = 0$. In the following we will find it more convenient to use two complex spinors ξ_{\pm}

$$\xi_{\pm} = \frac{1}{\sqrt{2}} (\epsilon^1 \pm i\epsilon^2). \tag{2.3}$$

Similar to the case presented in the previous subsection, we construct the $SU(3)$ invariant forms Ω, J, V

$$\begin{aligned} \Omega_{mnp} &= -\xi_+^\dagger \gamma_{mnp} \xi_-, \\ J_{mn} &= i\xi_+^\dagger \gamma_{mn} \xi_+ = -i\xi_-^\dagger \gamma_{mn} \xi_-, \\ V_m &= -\xi_+^\dagger \gamma_m \xi_+ = \xi_-^\dagger \gamma_m \xi_-. \end{aligned} \tag{2.4}$$

Note that in comparison to six-dimensional $SU(3)$ structures, in seven dimensions there also exists a globally defined vector field V . It is important to bear in mind that in general this vector is not a Killing direction and thus the manifold does not have the form of a direct product.

One can now show that Ω, J and V are all the possible independent combinations which one can construct and any other non-vanishing quantities can be expressed in terms of them. For example we have

$$\xi_-^\dagger \gamma_{mnp} \xi_+ = \bar{\Omega}_{mnp},$$

$$\xi_+^\dagger \gamma_{mnp} \xi_+ = \xi_-^\dagger \gamma_{mnp} \xi_- = i(J \wedge V)_{mnp}. \quad (2.5)$$

Furthermore, one can also show that the forms defined in (2.4) satisfy the seven-dimensional $SU(3)$ structure relations

$$\begin{aligned} J \wedge J \wedge J &= \frac{3i}{4} \Omega \wedge \bar{\Omega}, \\ \Omega \wedge J &= V \lrcorner J = V \lrcorner \Omega = 0, \end{aligned} \quad (2.6)$$

where the contraction symbol \lrcorner is defined in equation (A.4). Finally one can prove the following useful relations

$$\begin{aligned} V \lrcorner V &= 1, \\ J^m_i J^i_n &= -\delta_n^m + V^m V_n, \\ J_m^i \Omega_{\pm inp} &= \mp \Omega_{\mp mnp}, \\ \star \Omega^\pm &= \pm \Omega^\mp \wedge V, \\ \star(J \wedge V) &= \frac{1}{2} J \wedge J, \end{aligned} \quad (2.7)$$

where we have split the complex three-form Ω into its real and imaginary parts

$$\Omega = \Omega^+ + i\Omega^-. \quad (2.8)$$

Let us now see how to decompose the intrinsic torsion in $SU(3)$ modules. As before they are most easily defined from the differentials of the forms Ω , J and V . Generically we have [16, 22]

$$dV = RJ + \bar{W}_1 \lrcorner \Omega + W_1 \lrcorner \bar{\Omega} + A_1 + V \wedge V_1, \quad (2.9)$$

$$dJ = \frac{2i}{3} (c_1 \Omega - \bar{c}_1 \bar{\Omega}) + J \wedge V_2 + S_1 + V \wedge \left[\frac{1}{3} (c_2 + \bar{c}_2) J + \bar{W}_2 \lrcorner \Omega + W_2 \lrcorner \bar{\Omega} + A_2 \right], \quad (2.10)$$

$$d\Omega = c_1 J \wedge J + J \wedge T + \Omega \wedge V_3 + V \wedge [c_2 \Omega - 2J \wedge W_2 + S_2], \quad (2.11)$$

where the representatives of the 15 torsion classes are denoted by R , $c_{1,2}$, $V_{1,2,3}$, $W_{1,2}$, $A_{1,2}$, T and $S_{1,2}$. It is easy to read off the interpretation of the above torsion classes in terms of the $SU(3)$ structure group. There are three singlet classes R (real) and $c_{1,2}$ (complex), five vectors $V_{1,2,3}$ (real) and $W_{1,2}$ (complex), three 2-forms $A_{1,2}$ (real) and T (complex) and two 3-forms $S_{1,2}$.

Before concluding this section we should make more precise the relation between the $SU(3)$ and G_2 structures on a seven dimensional manifold. Obviously, as $SU(3) \subset G_2$, an $SU(3)$ structure automatically defines a G_2 structure on the manifold. In fact, an $SU(3)$ structure on a seven-dimensional manifold implies the existence of two independent G_2 structures whose intersection is precisely the $SU(3)$ structure. Concretely, using the spinor ϵ_1 and ϵ_2 defined above we can construct the two G_2 forms φ^\pm

$$\begin{aligned} (\varphi^+)_{mnp} &\equiv 2i\epsilon_1 \gamma_{mnp} \epsilon_1, \\ (\varphi^-)_{mnp} &\equiv 2i\epsilon_2 \gamma_{mnp} \epsilon_2. \end{aligned} \quad (2.12)$$

The relation to the $SU(3)$ structure is now given by

$$\varphi^\pm = \pm\Omega^- - J \wedge V. \tag{2.13}$$

Throughout this paper it will sometimes be useful to use the $SU(3)$ forms and sometimes the G_2 forms but we should keep in mind that the two formulations are equivalent.

2.3 Mass hierarchies

When the torsion on the internal manifold and the fluxes vanish, the holonomy group directly determines the amount of supersymmetry preserved in the vacuum. This is not the case with non-trivial G structures and/or fluxes where the amount of supersymmetry in the vacuum is not uniquely determined by the structure/holonomy group of the manifold. It should nevertheless be kept in mind that the amount of supersymmetry of the effective action is not unrelated to the structure group. In particular, the existence of globally defined spinors on the internal manifold allows us to define four-dimensional supercharges and therefore constitute a sufficient condition for supersymmetry of the effective action. Even though in general the situation can be more complicated we will assume that such supercharges, which are related to the globally defined spinors, are the only ones which survive in four dimensions and so the amount of supersymmetry of the effective action is given directly in terms of the structure group of the internal manifold.¹ Consequently, we will consider that M-theory compactifications on seven-dimensional manifolds with $SU(3)$ structure lead to an $\mathcal{N} = 2$ supergravity theory in four dimensions² while the vacuum may preserve $\mathcal{N} = 2$ or $\mathcal{N} = 1$ supersymmetry or even break it completely depending on which torsion classes (and fluxes) are turned on. This may be understood from the fact that when there are more than one internal spinors on the manifold they may satisfy different differential relations according to what torsion classes are present and so may correspond to different eigenvalues of the Dirac operator. Consider decomposing the eleven-dimensional gravitino in terms of the globally defined spinors on the internal manifold. Then the four-dimensional gravitini may have varying masses and there will appear mass hierarchies throughout the four-dimensional low-energy field spectrum. If the mass scales are well separated we can consider that only the lowest mass states are excited and so it is clear that in such a vacuum only a fraction of the original amount of supersymmetry is preserved. We will present such an example in section 5.4.2 where it will become clear that one of the two gravitini will become massive in the vacuum and thus supersymmetry will be spontaneously broken from $\mathcal{N} = 2$ to $\mathcal{N} = 1$.

¹We thank Nikolaos Prezas for pointing this out. For a recent discussion of this we refer the reader to [31].

²Strictly speaking, as manifolds with G_2 structure are known to have in fact $SU(2)$ structure [32], the effective action in four dimensions would be that of an $\mathcal{N} = 4$ supergravity. However, as $SU(2)$ structures in seven-dimensions are much less tractable than $SU(3)$ ones, we shall consider that the additional spinors lead to massive particles and we shall ignore them right from the beginning. In fact we shall see in sections 4 and 5 that for some seven-dimensional coset manifolds the $SU(2)$ structure is not compatible with the symmetries of the coset. As the lower mass states are associated with modes on the coset which obey the coset symmetries it is clear that such cases create a hierarchy between the four globally defined spinors effectively leading to a manifold with less globally defined spinors.

3. The reduction

The theory we will be considering is the low energy limit of M-theory that is eleven-dimensional supergravity. The bosonic action of the theory as well as the relevant gravitino terms are given by [33]

$$\begin{aligned}
 S_{11} = \frac{1}{\kappa_{11}^2} \int \sqrt{-g_{11}} d^{11} X & \left[\frac{1}{2} \hat{\mathcal{R}}_{11} - \frac{1}{2} \bar{\Psi}_M \hat{\Gamma}^{MNP} \hat{D}_N \Psi_P - \frac{1}{4!} \hat{F}_{MNPQ} \hat{F}^{MNPQ} \right. \\
 & + \frac{1}{2} \frac{1}{(12)^4} \epsilon^{LMNPQRSTUVW} \hat{F}_{LMNP} \hat{F}_{QRST} \hat{C}_{UVW} \\
 & \left. - \frac{3}{4(12)^2} (\bar{\Psi}_M \hat{\Gamma}^{MNPQRS} \Psi_N + 12 \bar{\Psi}^P \hat{\Gamma}^{QR} \Psi^S) F_{PQRS} \right]. \quad (3.1)
 \end{aligned}$$

The field spectrum of the theory contains the eleven-dimensional graviton \hat{g}_{MN} , the three-form \hat{C}_{MNP} and the gravitino, $\hat{\Psi}_P$. The indices run over eleven dimensions $M, N, \dots = 0, 1, \dots, 10$. For gamma matrix and epsilon tensor conventions see the Appendix. κ_{11} denotes the eleven-dimensional Planck constant which we shall set to unity henceforth thereby fixing our units.

In this section we will consider this theory on a space which is a direct product $M_{11} = M_4 \times K_7$ with the metric Ansatz

$$ds_{11}^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(x, y) dy^m dy^n, \quad (3.2)$$

where x denotes co-ordinates in four-dimensions and y are the co-ordinates on the internal compact manifold. The first thing to note is that this Ansatz is not the most general Ansatz possible for a metric as we have not included as possible dependence of the four-dimensional metric on the internal co-ordinates that is usually referred to as a warp factor. There are many compactifications that can consistently neglect such a warp factor because either a warp factor is not induced by the flux or it can be perturbatively ignored if the internal volume is large enough. Including such a warp factor is a difficult proposition for an action compactification because it can, and generally will, be a function of the four-dimensional moduli³. For now we will proceed with an unwarped Ansatz bearing in mind that this is only consistent for certain compactifications.

The four-dimensional effective theory will be an $\mathcal{N} = 2$ gauged supergravity. These type of theories have been studied extensively in the literature, see [34–39] and references within, and this work will be useful as a guide for the compactification. In the upcoming sections we will derive most of the quantities necessary to specify this theory. The kinetic terms for the low energy fields will be derived from the Ricci scalar and the kinetic term for the three-form. The prepotentials can then be derived from the four-dimensional gravitini mass matrix.

³This is not a problem when looking for solutions as they only probe the vacuum and are insensitive to moduli dynamics.

3.1 The Ricci scalar

As is well known, the metric on the compactification manifold is not rigid and its fluctuations can be written in terms of scalar fields in the effective low-energy theory. Important constraints on the spectrum and kinetic terms for these scalar fields come from the fact that they should form a four-dimensional $\mathcal{N} = 2$ supergravity. Compactifications of type II supergravities from ten to four dimensions on Calabi-Yaus naturally lead to such a supergravity. In this section we will show that it is possible to keep an analogy with these compactifications for the case of M-theory on $SU(3)$ structure manifolds that we are considering. A similar approach was adopted in [23] and we will closely follow their results.

3.1.1 The induced metric variations

Having $SU(3)$ structure on a manifold is a stronger condition than having a metric. Infact the $SU(3)$ structure induces a metric on the manifold that we can write in terms of the invariant forms as

$$\begin{aligned}
 g_{ab} &\equiv |s|^{-\frac{1}{9}} s_{ab} \\
 s_{ab} &\equiv \frac{1}{16} \left[\frac{1}{4} (\Omega_{amn} \bar{\Omega}_{bpq} + \bar{\Omega}_{amn} \Omega_{bpq}) + \frac{1}{3} V_a V_b J_{mn} J_{pq} \right] J_{rs} V_t \epsilon^{mnpqrst}.
 \end{aligned}
 \tag{3.3}$$

This expression for the metric can be checked by performing the contractions on the right-hand-side using the appropriate $SU(3)$ structure identities. Clearly, as the metric is determined uniquely in terms of the structure forms, all the metric fluctuations can be treated as fluctuations of the structure forms. The converse however is not true as it is possible that different structure forms give rise to the same or equivalent metrics. Therefore, when expressing the metric variations in terms of changes in the structure forms one has to take care not to include the spurious variations as well.

Varying the formula above we can write the metric deformations as

$$\begin{aligned}
 \delta g_{ab} &= \frac{1}{8} \delta \Omega_{(a}{}^{mn} \bar{\Omega}_{b)mn} + \frac{1}{8} \Omega_{(a}{}^{mn} \delta \bar{\Omega}_{b)mn} + 2V_{(a} \delta V_{b)} + V_a V_b (J_{\perp} \delta J) + J_{(a}{}^m \delta J_{b)m} \\
 &\quad + V^m V_{(a} J_{b)}^n \delta J_{mn} - \frac{1}{3} \left(\frac{1}{4} \delta \Omega_{\perp} \bar{\Omega} + \frac{1}{4} \Omega_{\perp} \delta \bar{\Omega} + J_{\perp} \delta J \right) g_{ab}.
 \end{aligned}
 \tag{3.4}$$

Note that this is very similar to normal Calabi–Yau compactifications where the metric variations were expressed in terms of Kähler class and complex structure deformations. Keeping the terminology we will refer to the scalar fields associated with δJ and $\delta \Omega$ as Kähler moduli and complex structure moduli respectively. Furthermore we will denote the scalar associated to δV as the dilaton in complete analogy to the type IIA compactifications.

Before starting the derivation of the kinetic terms associated to the metric deformations discussed above we mention that the metric variations can be dealt with more easily in terms of the variations of either of the two G_2 structures which can be defined on seven-dimensional manifolds with $SU(3)$ structure (2.13)

$$\delta g_{ab} = \frac{1}{2} \varphi_{(a}^{\pm mn} \delta \varphi_{b)mn}^{\pm} - \frac{1}{3} (\varphi^{\pm}_{\perp} \delta \varphi^{\pm}) g_{ab}.
 \tag{3.5}$$

Therefore, for each of the G_2 structures the formula coincides with the metric variations on a manifold with G_2 structure [12].

3.1.2 The Ricci scalar reduction

Let us now see explicitly how to derive the kinetic terms for the moduli fields described above. As they are metric moduli, their kinetic terms should appear from the compactification of the eleven-dimensional Ricci scalar. The explicit calculation is presented in Appendix B and here we will only outline the main steps before stating the final result. We should also mention that during this process we are mainly interested in the fate of the scalar fields which appear as fluctuations of the metric on the internal manifold and therefore we shall not discuss the vector field (graviphoton), which also arises from the metric, as we expect that its kinetic term is the standard one.

For now we do not decompose Ω and J into their four-dimensional scalar components but with the vector V we write

$$V(x, y) \equiv e^{\hat{\phi}(x)} z(y), \tag{3.6}$$

where z is the single vector we have on the internal manifold from the $SU(3)$ structure requirements. Note that it is still V and not z that features in the $SU(3)$ relations (2.6). The difference between V and z can be understood as V is the $SU(3)$ vector which also encodes the possible deformations of the manifold, while z is only a basis vector in which we expand V . Therefore, the factor $e^{\hat{\phi}}$ encodes information about the deformations associated to the vector V . This is completely analogous to the compactification of eleven-dimensional supergravity on a circle to type IIA theory and in order to continue this analogy we shall call the modulus in equation (3.6) the dilaton. Let us further define a quantity which in the case where the compactification manifold becomes a direct product of a six-dimensional manifold (with $SU(3)$ structure) and a circle, plays the role of the volume of the six-dimensional space

$$\mathcal{V}_6 \equiv e^{-\hat{\phi}} \mathcal{V}, \tag{3.7}$$

where \mathcal{V} is the volume of the full seven-dimensional space

$$\mathcal{V} \equiv \int \sqrt{g_7} = \frac{1}{6} \int J \wedge J \wedge J \wedge V. \tag{3.8}$$

To see the use of this quantity, note that due to the first relation in (2.6), a scaling of the three-form Ω automatically induces a change in the volume. Thus, scalings of Ω would have the same effect as appropriate scalings of J and in order not to count the same degree of freedom twice we shall define

$$e^{\frac{1}{2}K_{cs}} \Omega^{cs} \equiv \frac{1}{\sqrt{8}} \Omega (\mathcal{V}_6)^{-\frac{1}{2}}, \tag{3.9}$$

where we have also introduced the Kähler potential for the complex structure deformations, K_{cs} , extending the results of [23, 40, 41]

$$K_{cs} \equiv -\ln(\|\Omega^{cs}\| \mathcal{V}_6) = -\ln i \langle \Omega^{cs} | \bar{\Omega}^{cs} \rangle \equiv \int \Omega^{cs} \wedge \bar{\Omega}^{cs} \wedge z. \tag{3.10}$$

It is easy to check that rescalings of Ω precisely cancel the corresponding variation of \mathcal{V}_6 on the RHS of equation (3.9) and hence Ω^{cs} defined on the LHS stays unchanged. In this way we have managed to decouple the volume modulus from the form Ω . The relation (3.9)

deserves one more explanation. The additional factor on the LHS, $\exp \frac{1}{2}K_{cs}$ has been introduced in order to describe by Ω^{cs} the exact analogue of the Calabi–Yau holomorphic 3-form whose norm precisely gives the Kähler potential of the complex structure deformations.

One more comment is in order here. As explained before, not all the variations of the structure forms induce valid metric deformations. In particular the definition of the 3-form Ω (2.4) allows for an arbitrary phase which would subsequently drop out from the metric variations (3.4). In order to make sure that such variations are not introduced as degree of freedom we should “gauge” these phase transformation for Ω . Given the Kähler potential (3.10) and the definition (3.9) it is not hard to see that Kähler transformations, which correspond to scalings of Ω^{cs} by some function which is holomorphic in the complex structure moduli, precisely correspond to phase variations of Ω . Therefore, the covariant derivative for the “gauged” phase transformations of Ω should precisely be the Kähler covariant derivative

$$D_\mu \Omega \equiv \partial_\mu \Omega + \frac{1}{2} \partial_\mu K_{cs} \Omega = \sqrt{8\mathcal{V}_6} e^{\frac{1}{2}K_{cs}} (\partial_\mu \Omega^{cs} + \partial_\mu K_{cs} \Omega^{cs}) \equiv \sqrt{8\mathcal{V}_6} e^{\frac{1}{2}K_{cs}} D_\mu \Omega^{cs}. \quad (3.11)$$

Finally we note that we have to take into account the usual Weyl rescalings in order to arrive to the four-dimensional Einstein frame

$$\begin{aligned} g_{\mu\nu} &\rightarrow \mathcal{V}^{-1} g_{\mu\nu}, \\ g_{mn} &\rightarrow e^{-\frac{2}{3}\hat{\phi}} g_{mn}. \end{aligned} \quad (3.12)$$

Following the above steps one can derive the (linearised) variation of the Ricci scalar under the metric fluctuation (3.4). The calculation is presented in the appendix and here we recall the final result

$$\begin{aligned} \int \sqrt{-g_{11}} d^{11} X \frac{1}{2} \mathcal{R}_{11} &= \int \sqrt{-g_4} d^4 x \left[\frac{1}{2} \mathcal{R}_4 - \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} e^{2\phi} \mathcal{V}^{-1} \int \sqrt{g_7} \mathcal{R}_7 \right. \\ &\quad - \frac{1}{8} e^{-\hat{\phi}} e^{K_{cs}} \int \sqrt{g_7} d^7 y D_\mu \Omega^{cs} \lrcorner D^\mu \bar{\Omega}^{cs} \\ &\quad \left. - \frac{1}{4} \mathcal{V}_6^{-1} e^{-\hat{\phi}} \int \sqrt{g_7} d^7 y \partial_\mu J \lrcorner \partial^\mu J \right], \end{aligned} \quad (3.13)$$

where we have also defined the four-dimensional dilaton

$$\phi \equiv \hat{\phi} - \frac{1}{2} \ln \mathcal{V}_6. \quad (3.14)$$

The important thing to notice on this result is that the metric fluctuations have naturally split into the dilaton, the J and Ω^{cs} variations with separate kinetic terms. Moreover, due to the dependence of $\sqrt{g_7}$ on the dilaton, it can be seen that the all the dilaton factors drop out from the kinetic terms of the Kähler and complex structure moduli. Therefore, this result is very much like the one for usual type IIA compactifications on Calabi–Yau manifolds with the notable difference that a potential for the moduli appears due to the fact that manifolds with $SU(3)$ structure are in general no longer Ricci flat.

3.2 Four-dimensional field content and kinetic terms

In this section we will complete the kinetic terms for the low energy scalar field spectrum by reducing the three-form field \hat{C}_3 . These scalar fields pair up with the geometrical moduli into $\mathcal{N} = 2$ multiplets. We will however ignore the presence of additional fields, like gauge fields, which are expected to have similar kinetic terms to the gauge fields coming from type IIA compactifications.

3.2.1 Reduction of the three-form

As we have seen in the previous subsection, the compactification of the gravitational sector of M-theory on seven-dimensional manifolds with $SU(3)$ structure resembles very much the corresponding compactifications of type IIA theory on Calabi–Yau manifolds. Therefore we will find it useful to continue this analogy at the level of the matter fields and so we will first decompose the 3-form \hat{C}_3 along the vector direction which is featured in the seven-dimensional manifolds with $SU(3)$ structure under consideration. Consequently we write

$$\hat{C}_3 = C_3 + B_2 \wedge z, \tag{3.15}$$

where C_3 is assumed to have no component along z , ie $C_{3 \lrcorner z} = 0$. As expected, in the type IIA picture C_3 will correspond to the RR 3-form, while B_2 represents the NS-NS 2-form field. Then compactifying the eleven-dimensional kinetic term, taking care to perform the appropriate Weyl rescalings (3.12), we arrive at

$$\begin{aligned} & \int \sqrt{-g_{11}} d^{11} X \left[-\frac{1}{4} \hat{F} \lrcorner \hat{F} \right] \\ &= \int \sqrt{-g_4} d^4 x \left[-\frac{1}{4} e^{2\phi} e^{-\hat{\phi}} \int \sqrt{g_7} d^7 y \partial_\mu C_{3 \lrcorner} \partial^\mu C_3 - \frac{1}{4} \mathcal{V}_6^{-1} e^{-\hat{\phi}} \int \sqrt{g_7} d^7 y \partial_\mu B_{2 \lrcorner} \partial^\mu B_2 \right]. \end{aligned} \tag{3.16}$$

One immediately notices that the kinetic term for fluctuations of the B_2 -field along the internal manifold is the same as the kinetic term for the fluctuations of the fundamental form J . Therefore we see that these fluctuations pair up into the complex field

$$T \equiv B_2 - iJ. \tag{3.17}$$

In order to analyse the four-dimensional effective action we have to specify which are the modes we want to preserve in a Kaluza-Klein truncation. In general one restricts to the lowest mass modes, but in the case at hand this is a hard task partly due to the big uncertainties regarding the spectrum of the Laplace operator on forms for arbitrary manifolds with $SU(3)$ structure. The best thing we can do is to use our knowledge from other similar cases where the structure of four-dimensional theory was derived [23, 40, 42–44], as well as the close analogy to the type IIA compactifications and postulate the existence of a set of forms in which to expand the fluctuations we have discussed so far. For the moment these forms are quite arbitrary, but for specific cases it should be possible to derive some of their most important properties. In fact we shall see such examples in sections 4 and 5 where explicit examples of manifolds with $SU(3)$ structure will be discussed. Therefore we consider a set of two-forms, ω_i , with dual four-forms, $\tilde{\omega}^i$ which

satisfy

$$\int \omega_i \wedge \tilde{\omega}^j \wedge z = \delta_i^j. \tag{3.18}$$

Furthermore we introduce three-forms (α_A, β^A) which obey

$$\begin{aligned} \int \alpha_A \wedge \beta^B \wedge z &= \delta_A^B, \\ \int \alpha_A \wedge \alpha_B \wedge z &= \int \beta^A \wedge \beta^B \wedge z = 0. \end{aligned} \tag{3.19}$$

Anticipating that we expand the structure variations in these forms we also consider them to be compatible with the $SU(3)$ structure relations (2.6) and (2.7)

$$\begin{aligned} \omega_i \wedge \alpha_A &= \omega_i \wedge \beta^A = 0, \\ z \lrcorner \omega_i &= z \lrcorner \alpha_A = z \lrcorner \beta^A = 0. \end{aligned} \tag{3.20}$$

These forms can in general depend on all seven internal coordinates and not be closed. The index ranges are not necessarily topological but should correspond to the number of generalised calibrated submanifolds in the internal manifold [40, 42–44].

Given the forms defined above we should expand all the fluctuations and interpret the coefficients as the four-dimensional degrees of freedom. Consequently we write for the metric variations

$$\begin{aligned} J(x, y) &= v^i(x) \omega_i(y), \\ \Omega^{cs}(x, y) &= Z^A(x) \alpha_A(y) - F_A(Z(x)) \beta^A(y), \end{aligned} \tag{3.21}$$

where we have already used the fact that the deformations of Ω span a special-Kähler manifold and therefore can be written as above, where F_A is a holomorphic function of the complex coordinates Z^A , which is also homogeneous of degree one in Z^A . From the four-dimensional perspective v^i are real scalar fields which we will refer to as Kähler moduli. Z^A on the other hand are not all independent and we shall consider as the true degrees of freedom the quantities $z^a = Z^a/Z^0$, where the index a runs over the same values as the index A , except for the value 0. For the matter fields we take

$$\begin{aligned} B_2(x, y) &= \mathring{B}_2(y) + \tilde{B}_2(x) + b^i(x) \omega_i(y), \\ C_3(x, y) &= \mathring{C}_3(y) + \tilde{C}_3(x) + A^i(x) \wedge \omega_i(y) + \xi^A(x) \alpha_A(y) - \tilde{\xi}_A(x) \beta^A(y). \end{aligned} \tag{3.22}$$

Note that in the above decomposition we have allowed for a background value for B_2 and C_3 which we denoted \mathring{B}_2 and \mathring{C}_3 respectively. These values should be understood as giving rise to the flux terms for the field strengths of B_2 and C_3 and therefore they should not be globally well defined over the internal manifold. We will postpone their discussion until the next section when we deal with background fluxes. Note that B_2 can not be expanded along the z direction as it already comes from a three-form with one leg along z , while C_3 was assumed not to have any component along z cf equation (3.15). The fields b^i , ξ^A and $\tilde{\xi}_A$ are scalar fields in four dimensions and they will be important for our following discussion. Moreover, $\tilde{B}_2(x)$ is a four-dimensional two-form which, in the absence of fluxes, can be dualised into an axion $b(x)$. Here however we will not perform this dualization as in the

$g_{\mu\nu}, A^0$	gravitational multiplet
$\xi^0, \tilde{\xi}_0, \phi, \tilde{B}_2$	universal tensor-multiplet
b^i, v^i, A^i	vector multiplets
ξ^a, ξ_a, z^a	hypermultiplets

Table 1: Table showing the $\mathcal{N} = 2$ multiplets.

examples we present in sections 4.2 and 5.4 the \tilde{B} -field will be massive in four dimensions and therefore we will keep it as a member of “the universal” tensor-multiplet. $\tilde{C}_3(x)$ is a three-form which carries no degree of freedom in four dimensions and is dual only to some constant, but its dualisation in four dimensions requires more care. As explained before, we shall not deal with the vector fields A^i here as their couplings are expected to be similar to the type IIA compactifications. Also we shall neglect other vector degrees of freedom which arise from the isometries of the internal manifold and leave their proper treatment for another project.

We will also find it useful to introduce at this level one more notation. As we are mostly interested in the scalar fields in the theory we will denote all the fluctuations of \hat{C}_3 which give rise to scalar fields in four dimensions by \hat{c}_3 . Just from its definition we can see that this is a three-form on the internal manifold. In terms of the expansions above it takes the form

$$\hat{c}_3(x, y) = b^i(x)(\omega_i \wedge z)(y) + \xi^A(x)\alpha_A(y) - \tilde{\xi}_A(x)\beta^A(y). \tag{3.23}$$

Finally, as we expect that the low energy effective action is a $N = 2$ (gauged) supergravity, the light fields should assemble into $N = 2$ multiplets. This is briefly reviewed in table 1. As mentioned before, the internal parts of the two form B , and the fundamental form J combine themselves into a complex field

$$T(x, y) \equiv B_2(x, y) - iJ(x, y) = t^i(x)\omega_i(y) \equiv (b^i(x) - iv^i(x))\omega_i(y), \tag{3.24}$$

which will become the scalar components of the $N = 2$ vector multiplets. The associated Kähler potential is again similar to the one in type IIA theory

$$K_t = -\ln \frac{1}{6} \int J \wedge J \wedge J \wedge z = -\ln \mathcal{V}_6. \tag{3.25}$$

As we expect from the structure of $\mathcal{N} = 2$ supergravity theories as well as from the analogy to type IIA compactifications [23, 40], the fields t^i span a special Kähler geometry with a cubic prepotential $\mathcal{F} = -\frac{1}{6} \frac{\mathcal{K}_{ijk} t^i t^j t^k}{t^0}$, where \mathcal{K}_{ijk} are the analogue of the triple intersection numbers

$$\mathcal{K}_{ijk} = \int \omega_i \wedge \omega_j \wedge \omega_k \wedge z. \tag{3.26}$$

The symplectic sections are given by $X^I = (t^0, t^i)$ and $\mathcal{F}_I = \partial_I \mathcal{F}$ with $t^0 = 1$. Indeed, one can easily check, using the expansion (3.21) that the Kähler potential above derives from the general $\mathcal{N} = 2$ formula $K = -\ln i(X^I \bar{\mathcal{F}}_I - \bar{X}^I \mathcal{F}_I)$.

It is interesting to note that while in type IIA compactifications with fluxes only charged hypermultiplets can appear, in the case of M-theory compactified on seven-dimensional manifolds with $SU(3)$ structure one can also obtain charged vector multiplets as also remarked in [27]. Indeed it is not hard to see that provided $\int d\omega_{i\perp}(\omega_j \wedge z) \equiv k_{ij}$ does not vanish, the kinetic term for the three-form \hat{C}_3 in eleven dimensions generates a coupling of the type $k_{ij}b^iA^j$ in the low energy effective action which precisely uncovers the fact that the scalars in the vector multiplets become charged.

3.3 Flux and gravitino mass matrix

So far we have only discussed the kinetic terms of the various fields which appear in the low energy theory and we have seen that their structure is very much like in type IIA compactifications. We will now turn to study the effect of the non-trivial structure group and of turning on fluxes. The only background fluxes which can be turned on in M-theory compactifications and which are compatible with four-dimensional Lorenz invariance can be written as

$$\left[\hat{F}_4\right]_{\text{Background}} = f\eta_4 + \mathcal{G}. \tag{3.27}$$

Here f is known as Freund-Rubin parameter where η_4 is the four-dimensional volume form and \mathcal{G} is the four-form background flux which can locally be written as

$$\mathcal{G} = d\hat{C}_3(y), \tag{3.28}$$

where $\hat{C}_3(y)$ is the background part of the three-form field \hat{C}_3 which was defined in equation (3.22). As observed in the literature [9, 12, 45], the Freund-Rubin flux is not the true constant parameter describing this degree of freedom. Rather one has to consider the flux of the dual seven-form field strength \hat{F}_7

$$\hat{F}_7 = d\hat{C}_6 + \frac{1}{2}\hat{C}_3 \wedge \hat{F}_4, \tag{3.29}$$

which should now be the true dual of the Freund-Rubin flux. As can be seen the \hat{F}_7 flux also receives a contribution from the ordinary \hat{F}_4 flux. Therefore, in general, the Freund-Rubin flux parameter is given by

$$f = \frac{1}{\mathcal{V}} \left(\lambda + \frac{1}{2} \int \hat{c}_3 \wedge \mathcal{G} + \frac{1}{2} \int \hat{c}_3 \wedge d\hat{c}_3 \right), \tag{3.30}$$

where λ is a constant which parameterizes the 7-form flux.

On top of these fluxes which can be turned on for the matter fields one has to consider the torsion of the internal manifold with $SU(3)$ structure which is also known as “metric flux”. The effects of the torsion can be summarised as follows. We have already seen that the compactification of the Ricci scalar contains a piece due to the non-vanishing scalar curvature of the internal manifold. This is entirely due to the torsion as manifolds with $SU(3)$ holonomy are known to be Ricci flat. Moreover, a non-trivial torsion is associated with non-vanishing exterior derivatives of the structure forms. If we insist that we expand the fluctuations of these structure forms as in equation (3.21) it is clear that the expansion

forms cannot be closed. Therefore, the presence of torsion forces us to perform the field expansions in forms which are no longer closed. Such forms will induce in the field strength of the three-form \hat{C}_3 terms which are purely internal and which are – from this point of view — indistinguishable from the normal fluxes and so the flux in (3.27) is modified to be the full field strength expression

$$y\hat{F}_4 = f\eta_4 + \mathcal{G} + d\hat{c}_3, \quad (3.31)$$

where the derivative should be understood as the exterior derivative on the seven-dimensional manifold. However such “induced” fluxes are not constant, but they depend on the scalar fields which arise from \hat{C}_3 . It is also worth noting at this point that provided these scalar fields are fixed at a non-vanishing value in the vacuum, these vacuum expectation values will essentially look like fluxes for \hat{F}_4 in that specific vacuum. We will use this fact later on when we discuss moduli stabilization.

As mentioned before, the effect of the fluxes and torsion is to “gauge” the $\mathcal{N} = 2$ supergravity theory and induce a potential for the scalar fields. These effects can be best studied in the gravitino mass matrix to which we now turn. In an $\mathcal{N} = 1$ supersymmetric theory, the gravitino mass is given by the Kahler potential and superpotential, while in an $\mathcal{N} = 2$ theory we have a mass matrix which is constructed out of the Killing prepotentials (electric and magnetic) that encode information about the gaugings in the hyper-multiplet sector. Moreover, the same gravitino mass matrix appears in the supersymmetry transformations of the four-dimensional gravitini and therefore its value in the vacuum gives information about the amount of supersymmetry which is preserved in that particular case. This can also be understood from the fact that unbroken supersymmetry requires vanishing physical masses⁴ for the gravitino and so, non-zero eigenvalues of the gravitino mass matrix in the vacuum imply partial or complete spontaneous supersymmetry breaking. In the case of partial supersymmetry breaking of an $\mathcal{N} = 2$ theory, the superpotential and D-terms of the resulting $\mathcal{N} = 1$ theory are completely determined by the $\mathcal{N} = 2$ mass matrix.

In a compactification from a higher-dimensional theory there are several ways to determine the gravitino mass matrix in the four-dimensional theory. If we have explicit knowledge of the four-dimensional degrees of freedom we can derive the complete bosonic action and from the potential and gaugings derive the $\mathcal{N} = 2$ Killing prepotentials. Alternatively one can directly perform a computation in the fermionic sector and directly derive the gravitino mass matrix or compactify the higher dimensional supersymmetry transformations. The advantage of the last two methods is that one obtains a generic formula for the mass matrix in terms of integrals over the internal manifold without explicit knowledge of the four-dimensional fields. Once these fields are identified in some expansion of the higher-dimensional fields one can obtain an explicit formula for the mass matrix which should also be identical to the one obtained from purely bosonic computations.

In the following we choose to determine the gravitino mass matrix by directly identifying all the possible contributions to the gravitino mass from eleven dimensions. For

⁴In AdS space, the mass parameter which appears in the Lagrangian is not the true mass of a particle. Therefore we use the terminology *physical mass* in order to distinguish the true mass from the parameter which appears in the Lagrangian.

this we will first have to identify the four-dimensional gravitini. Recall from section 2.2 that on a seven-dimensional manifold with $SU(3)$ structure one can define two independent (Majorana) spinors which we have denoted $\epsilon_{1,2}$. Then, we consider the Ansatz

$$\hat{\Psi}_\mu = \mathcal{V}^{-\frac{1}{4}} (\psi_\mu^1 \otimes \epsilon_1 + \psi_\mu^2 \otimes \epsilon_2) , \quad (3.32)$$

where $\psi^{1,2}$ are the four-dimensional gravitini which are Majorana spinors and the overall normalisation factor is chosen in order to reach canonical kinetic terms in four-dimensions. It is more customary to work with gravitini which are Weyl spinors in four dimensions and therefore we decompose $\psi^{1,2}$ above as

$$\psi_\mu^\alpha = \frac{1}{2} (\psi_{+\mu}^\alpha + \psi_{-\mu}^\alpha) , \quad (3.33)$$

where $\alpha, \beta = 1, 2$ and the chiral components of four-dimensional gravitini satisfy

$$\gamma_5 \psi_{\pm\mu}^\alpha = \pm \psi_{\pm\mu}^\alpha . \quad (3.34)$$

Then compactifying the eleven-dimensional gravitino terms in (3.1) and performing the appropriate Weyl rescalings (3.12) we arrive at the four-dimensional action

$$\tilde{S}_{\psi_\mu} = \int_{\mathcal{M}_4} \sqrt{-g} \left[-\bar{\psi}_{+\mu}^\alpha \gamma^{\mu\nu\rho} D_\nu \psi_{+\rho}^\alpha + S_{\alpha\beta} \bar{\psi}_{+\mu}^\alpha \gamma^{\mu\nu} \psi_{-\nu}^\beta + \text{c.c.} \right] . \quad (3.35)$$

The main steps in deriving the mass matrix are presented in appendix C and for similar calculations we refer the reader to the existing literature [12, 23, 57] where similar calculations were performed. Equation (C.14), which is the final result for the gravitino mass matrix $S_{\alpha\beta}$, can be written as

$$\begin{aligned} S_{11} &= \frac{ie^{\frac{7}{2}\hat{\phi}}}{8\mathcal{V}^{\frac{3}{2}}} \left\{ \int_{\mathcal{M}_7} [dU^+ \wedge U^+ + 2\mathcal{G} \wedge U^+] + 2\lambda \right\} , \\ S_{22} &= \frac{ie^{\frac{7}{2}\hat{\phi}}}{8\mathcal{V}^{\frac{3}{2}}} \left\{ \int_{\mathcal{M}_7} [dU^- \wedge U^- + 2\mathcal{G} \wedge U^-] + 2\lambda \right\} , \\ S_{12} = S_{21} &= \frac{ie^{\frac{5}{2}\hat{\phi}}}{8\mathcal{V}^{\frac{3}{2}}} \int_{\mathcal{M}_7} [2i\mathcal{G} \wedge \Omega^+ + 2id\hat{c} \wedge \Omega^+ - 2dJ \wedge \Omega^+ \wedge z] . \end{aligned} \quad (3.36)$$

Here \mathcal{G} denotes the internal part of the background flux which was defined in equation (3.28), λ is the constant to which the three-form \tilde{C}_3 is dual in four dimensions and we have further introduced

$$U^\pm \equiv \hat{c}_3 + ie^{-\hat{\phi}} \phi^\pm = \hat{c}_3 \pm ie^{-\hat{\phi}} \Omega^- - iJ \wedge z , \quad (3.37)$$

where \hat{c}_3 denotes the purely internal value of the three-form field \hat{C}_3 which was defined in equation (3.23).

The diagonal terms in the mass matrix correspond to the gravitino masses for separate compactifications on the two G_2 structures. This follows from associating each of the four-dimensional gravitini with one of the two internal spinors in the G_2 forms (2.12). We can also read off the prepotentials, P^x and Q^x for the hypermultiplets and the Kähler potential, K , for the vector multiplets of the $\mathcal{N} = 2$ supergravity by comparing the mass matrix with

the general expression for an $\mathcal{N} = 2$ gauged supergravity [37–39]

$$S_{\alpha\beta} = \frac{ie^{\frac{1}{2}K}}{2} \sigma_{\alpha\beta}^x (P_A^x X^A - Q^{xA} F_A), \tag{3.38}$$

where P_A^x and Q^{xA} are the electric respectively magnetic prepotentials which depend on the hypermultiplets in the theory while (X^A, F_A) is a symplectic section which characterizes the special Kähler geometry of the vector multiplet scalars. Note that we have used the general formula for the $\mathcal{N} = 2$ gauged supergravity mass matrix which appears when both electric and magnetic gaugings are present. This is because we expect to have both type of gaugings which is in general signaled by the presence of massive tensor multiplets in the four-dimensional effective action. It is easy to infer that such massive tensors appear if one takes into account that the one-form z , used in the expansion (3.15), is not closed. Squaring the field strength which comes from this expansion, B_2 will pick up a mass proportional to $\int dz \wedge \star dz$.

Finally we note that in a generic vacuum the off diagonal components of the mass matrix are non-vanishing and therefore the gravitini as defined in equation (3.32) are not mass eigenstates. The masses of the two gravitini are then given by the eigenvalues of the mass matrix evaluated in the vacuum. If these masses are equal and the two gravitini physically massless then supersymmetry is preserved in the vacuum. However this is not the case in general and then one encounters partial (when one gravitino is physically massless) or total spontaneous supersymmetry breaking. We shall come back to this issue in section 5.

4. Preserving N=2 supersymmetry

In this section we will consider the case where the internal manifold is one that will preserve the full $\mathcal{N} = 2$ supersymmetry in the vacuum. We will begin by studying the constraints such a solution should satisfy in section 4.1, moving onto studying the form of the mass matrix for this solution in section 4.1.1. Finally in section 4.2 we will go through an explicit example of such a vacuum by considering the coset $SO(5)/SO(3)_{A+B}$.

4.1 N=2 solution

In this section we will classify the most general manifolds with $SU(3)$ structure that are solutions to M-theory that preserve $\mathcal{N} = 2$ supersymmetry with 4D spacetime being Einstein and admitting two Killing spinors. In order to study such solutions in full generality we allow for a warped product metric

$$ds_{11}^2 = e^{2A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(x, y) dy^m dy^n, \tag{4.1}$$

but will eventually show that the warp factor, $A(y)$, vanishes. This class of solutions has also been recently discussed in [22]. We look for solutions to the eleven-dimensional Killing spinor equation

$$\nabla_M \eta + \frac{1}{288} \left[\Gamma_M^{NPQR} - 8\delta_M^{[N} \Gamma^{PQR]} \right] \hat{F}_{NPQR} \eta = 0. \tag{4.2}$$

For the background field strength \hat{F}_{MNPQ} above we will consider the most general Ansatz compatible with four-dimensional Lorentz invariance. Therefore, the only non-vanishing components of \hat{F} are \hat{F}_{mnpq} and $F_{\mu\nu\rho\sigma} = f\epsilon_{\mu\nu\rho\sigma}$.

Given that the internal manifold has $SU(3)$ structure we know there exist at least two globally defined Majorana spinors and so we take a killing spinor Ansatz

$$\eta = \theta_1(x) \otimes \epsilon_1(y) + \theta_2(x) \otimes \epsilon_2(y). \tag{4.3}$$

Since we are looking for $N = 2$ solution we treat θ_1 and θ_2 as independent. This will lead to more stringent constraints than the $\mathcal{N} = 1$ case, where they may be related, which will make finding the most general solution straightforward. As we are looking for four-dimensional maximally symmetric spaces, the Killing spinors $\theta_{1,2}$ satisfy

$$\nabla_\mu \theta_i = -\frac{i}{2} \Lambda_1^i \gamma_\mu \gamma_5 \theta_i + \frac{1}{2} \Lambda_2^i \gamma_\mu \theta_i \quad (\text{no sum over } i), \tag{4.4}$$

where the index $i = 1, 2$ labels the two spinors. The integrability condition reads

$$R_{\mu\nu} = -3 \left[(\Lambda_1^i)^2 + (\Lambda_2^i)^2 \right] g_{\mu\nu}, \quad i = 1, 2, \tag{4.5}$$

and so one immediately sees that not all $\Lambda_{1,2}^i$ are independent, but have to satisfy

$$(\Lambda_1^1)^2 + (\Lambda_2^1)^2 = (\Lambda_2^2)^2 + (\Lambda_1^2)^2. \tag{4.6}$$

Now decomposing the Killing spinor equation into its external and internal parts we arrive at the following equations

$$\nabla_m \epsilon_{1,2} = \left(\frac{i}{12} e^{-4A} f \gamma_m \right) \epsilon_{1,2}, \tag{4.7}$$

$$0 = \left(\gamma_m^{npqr} \hat{F}_{npqr} - 8 \gamma^{pqr} \hat{F}_{mpqr} \right) \epsilon_{1,2}, \tag{4.8}$$

$$\left(\frac{i}{2} \Lambda_1^{1,2} \right) \epsilon_{1,2} = \left(\frac{1}{2} e^A \gamma^n \partial_n A + \frac{i}{6} e^{3A} f \right) \epsilon_{1,2}, \tag{4.9}$$

$$\left(\frac{1}{2} \Lambda_2^{1,2} \right) \epsilon_{1,2} = \left(-\frac{1}{288} e^A \gamma^{npqr} \hat{F}_{npqr} \right) \epsilon_{1,2}. \tag{4.10}$$

In order to classify this solution from the point of view of the $SU(3)$ structure we have find the corresponding non-vanishing torsion classes by computing the exterior derivatives of the structure forms. Using their definition in terms of the spinors (2.4) and applying the results above one finds

$$\begin{aligned} dV &= \frac{1}{3} f J, \\ dJ &= 0, \\ d\Omega &= -\frac{2i}{3} f \Omega \wedge V, \\ dA &= 0. \end{aligned} \tag{4.11}$$

The first thing to note is that the warp factor A is constant in this vacuum and therefore can be set to zero by a constant rescaling of the metric. The second thing to observe, comparing with equation (2.11), is that only the singlet classes R and c_2 are non-vanishing. Moreover, they are not independent, but proportional to each other as they can both be expressed in terms of the Freund-Rubin parameter f .

From equations (4.7) we can also determine the parameters $\Lambda_{1,2}^i$, which determine the value of the cosmological constant, which are given by

$$\begin{aligned}\Lambda_1^1 &= \Lambda_1^2 = \frac{f}{3}, \\ \Lambda_2^1 &= \Lambda_2^2 = 0.\end{aligned}\tag{4.12}$$

The Killing spinor equations (4.7) also give constraints on the internal flux that imply it should vanish. However an easier way to see this is to consider the integral of the external part of the eleven-dimensional Einstein equation which reads

$$\int R_{(4)} + \frac{4}{3} \int f^2 + \frac{1}{72} \int \hat{F}_{mnpq} \hat{F}^{mnpq} = 0.\tag{4.13}$$

We see that substituting (4.12) we indeed recover $\hat{F}_{mnpq} = 0$. Since f is a constant, this means that the Bianchi Identity $dF = 0$ is indeed satisfied and so follow all the equations of motion.

Finally we note that in terms of the two G_2 structures φ^\pm , equations (4.11) can be recast into a simple form

$$d\varphi^\pm = \frac{2}{3} f \star \varphi^\pm,\tag{4.14}$$

which shows that both G_2 structures are in fact weak G_2 .

4.1.1 The mass of the gravitini

We can now use this solution to illustrate the discussion on the relation between the gravitini masses and supersymmetry and to check our form of the mass matrix. Inserting the solution just derived into the mass matrix we should find that the masses of the two gravitini degenerate and that they are both physically massless. Taking the solution (4.11) from the previous section the mass matrix (3.36) reads

$$\begin{aligned}S_{12} &= 0, \\ S_{11} = S_{22} &= \frac{-if e^{\frac{7}{2}\hat{\phi}}}{3\mathcal{V}^{\frac{1}{2}}},\end{aligned}\tag{4.15}$$

which indeed shows that the masses of the two gravitini are the same. To show that the two gravitini are physically massless we recall that in AdS space the physical mass of the gravitino is given by

$$m_{phys} = m_{3/2} - l,\tag{4.16}$$

where $m_{3/2}$ is the actual mass parameter which appears in the Lagrangian (in our case $|S_{11}|$), while l is the AdS inverse radius and is defined as

$$R = -12l^2,\tag{4.17}$$

with R the corresponding Ricci scalar.

In order to obtain the AdS radius correctly normalised we recall that the mass matrix (4.15) was obtained in the Einstein frame which differs from the frame used in the previous section by the Weyl rescaling (3.12). Inserting this into (4.5) we obtain the properly normalised AdS inverse radius

$$l = \frac{f e^{\frac{7}{2}\hat{\phi}}}{3\mathcal{V}^{\frac{1}{2}}}. \tag{4.18}$$

Note that here, as well as in equation (4.15), the fields $\hat{\phi}$ and \mathcal{V} should be replaced with their particular values which they have for this solution. Equation (4.18), together with (4.15), shows that the physical mass of the gravitini, (4.16), vanishes confirming our expectations that the vacuum determined in the previous section does indeed preserve $\mathcal{N} = 2$ supersymmetry.

4.2 The coset $SO(5)/SO(3)_{A+B}$

In order to see the above considerations at work we will now go through an explicit example of a manifold that satisfies the $N = 2$ solution discussed in the previous sections. The manifold we will consider is the coset space $SO(5)/SO(3)_{A+B}$. Cosets are particularly useful as examples of structure manifolds because the spectrum of forms that respect the coset symmetries is highly constrained. There are more details about cosets in general and about this particular coset in the appendix, or, for further reference we refer the reader to [46]. In this section we summarise the results and construct a basis of forms with which we can perform the compactification.

We begin by finding the most general symmetric two-tensor that respects the coset symmetries, this will be the metric on the coset and is given by

$$g = \begin{pmatrix} a & 0 & 0 & 0 & d & 0 & 0 \\ 0 & a & 0 & 0 & 0 & d & 0 \\ 0 & 0 & a & 0 & 0 & 0 & d \\ 0 & 0 & 0 & b & 0 & 0 & 0 \\ d & 0 & 0 & 0 & c & 0 & 0 \\ 0 & d & 0 & 0 & 0 & c & 0 \\ 0 & 0 & d & 0 & 0 & 0 & c \end{pmatrix}, \tag{4.19}$$

where all the parameters are real. The parameters of the metric are the geometrical moduli and we see that we have four real moduli on this coset. Note that there is a positivity domain $ac > d^2$. Having established the metric on the coset we can move on to find the structure forms. The strategy here is to find the most general one, two and three forms and then impose the $SU(3)$ structure relations on them. It is at this stage that we really see what the G structure of the coset is. This analysis is performed in the appendix and we find that the structure forms are given by

$$\begin{aligned} V &= e^{\hat{\phi}} z, \\ J &= v \omega, \\ \Omega &= \zeta_3 \alpha_0 + \zeta_4 \alpha_1 + \zeta_6 \beta^1 + \zeta_7 \beta^0, \end{aligned} \tag{4.20}$$

where the relations between the ζ s and the metric moduli are given in the appendix. The basis forms satisfy the differential relations

$$\begin{aligned}
 dz &= -\omega, \\
 d\omega &= 0, \\
 d\alpha_0 &= z \wedge \alpha_1, \\
 d\beta^0 &= -z \wedge \beta^1, \\
 d\alpha_1 &= 2z \wedge \beta^1 - 3z \wedge \alpha_0, \\
 d\beta^1 &= -2z \wedge \alpha_1 + 3z \wedge \beta^0.
 \end{aligned}
 \tag{4.21}$$

The structure forms (4.20) show that indeed the coset has exactly $SU(3)$ structure. In terms of the moduli classification we have been using it has a dilaton, one Kähler modulus and one complex structure modulus⁵ thus making up the four degrees of freedom in the metric. We also show in appendix 4.2 that scalar functions are in general not compatible with coset symmetries and therefore we conclude that for such compactifications no warp factor can appear.

4.2.1 Finding $\mathcal{N} = 2$ minima

In this section we want to find out if the potential which arises from the compactification on the coset above has a minimum where the geometric moduli are stabilised. In particular we wish to look for minima that preserve $\mathcal{N} = 2$ supersymmetry and correspond to the solution discussed in section 4.1. As usual, in a bosonic background, the condition for supersymmetry is the vanishing of the supersymmetry variations of the fermions. This is precisely what we used in the previous section and thus a supersymmetric solution should satisfy all the conditions derived there, and in particular (4.11). It is easy to see that the forms (4.20) obey

$$\begin{aligned}
 dV &= -\frac{e^{\hat{\phi}}}{v} J, \\
 dJ &= 0, \\
 d\Omega &= z \wedge [(-3\zeta_4)\alpha_0 + (\zeta_3 - 2\zeta_6)\alpha_1 + (2\zeta_4 - \zeta_7)\beta^1 + (3\zeta_6)\beta^0].
 \end{aligned}
 \tag{4.22}$$

Therefore these forms will in general not satisfy the solution constraints (4.11). Requiring them to match the solution gives a set of equations for the moduli that will exactly determine the value of the moduli in the vacuum. For the coset at hand these are easy to solve and the solution is given by

$$\begin{aligned}
 e^{\hat{\phi}} &= \frac{6^{\frac{1}{3}}\sqrt{42}}{14}\lambda^{\frac{1}{6}}, \\
 v &= \frac{6^{\frac{2}{3}}}{7}\lambda^{\frac{1}{3}}, \\
 \zeta_3 = -\zeta_6 = -i\zeta_4 = i\zeta_7 &= \frac{6}{49}(i-1)\sqrt{7\lambda},
 \end{aligned}
 \tag{4.23}$$

⁵As is expected from $\mathcal{N} = 2$ supergravity the parameters $\zeta_3, \zeta_4, \zeta_6$ and ζ_7 describe only two real degrees of freedom.

where we have replaced the Freund-Rubin flux f by true flux parameter λ from equation (3.30). Note that this solution fixes all the geometric moduli which is an important result for M-theory compactifications. It is important to stress however that ζ are not the true complex structure moduli, but are related to them by the rescaling (3.9). However, the complex structure moduli defined in (3.21), which can be most easily read off in special coordinates, do not depend on the rescalings of Ω and therefore, in our case the value of the single modulus is given by

$$z^1 = \frac{Z^1}{Z^0} = \frac{\zeta_4}{\zeta_3} = i \tag{4.24}$$

It can also be shown that the other scalar fields, which come from the expansion of the 3-form \hat{C}_3 , (3.22), in the forms (4.21) are also stabilised. A simple argument to support this statement is that non-vanishing values of these scalars would lead to a non-zero internal \hat{F}_4 flux at this vacuum solution due to the non-trivial derivative algebra the basis forms satisfy, (4.21), which in turn is ruled out by the supersymmetry conditions found in section 4.1. Hence, these scalar fields are forced by supersymmetry to stay at zero vacuum expectation value and therefore are fixed.

It is also worth observing one more thing regarding this solution. If we think in terms of the type IIA quantities we see that the Kähler modulus v and the dilaton $e^{\hat{\phi}}$ are not independent and choosing to stay in the supergravity approximation on type IIA side, ie take $v \gg 1$, drives the theory to the strong coupling regime which explains why such solutions can not be seen in the perturbative type IIA approach.

Finally we note that as the solution above is supersymmetric, the four-dimensional space-time is AdS with the AdS curvature which scales with λ as

$$l \sim \frac{1}{\lambda^{\frac{1}{6}}}. \tag{4.25}$$

Thus, in the large volume limit (ie $\lambda \gg 1$) the four-dimensional space-time approaches flat space.

5. Preserving $N = 1$ supersymmetry

In this section we will analyse the case where we only preserve $\mathcal{N} = 1$ supersymmetry in the vacuum. We will show that this occurs due to spontaneous partial supersymmetry breaking, much like in massive type IIA [23], and that it is possible to write an effective $\mathcal{N} = 1$ theory about this vacuum. We will derive the Kähler potential and superpotential for this theory and go through an explicit example of a manifold that leads to this phenomenon.

5.1 Spontaneous partial supersymmetry breaking

In section 3.3 we showed that for certain manifolds there is a mass gap between the two gravitini in the vacuum and if this is the case then the vacuum no longer preserves the full $\mathcal{N} = 2$ supersymmetry but rather spontaneously breaks to either $\mathcal{N} = 1$ or $\mathcal{N} = 0$ supersymmetry the former corresponding to one physically massless gravitino and the latter to no massless gravitini. In this section we will consider the case where the vacuum still preserves $\mathcal{N} = 1$ supersymmetry. With this a mass gap of the scale of supersymmetry

breaking, which is set by the vev of the scalars, appears throughout the spectrum and so we can consider specifying an effective $\mathcal{N} = 1$ theory that is composed of the lower mass states. The superpotential and the Kähler potential for this theory will then be given by the mass of the physically massless gravitino as is usual for $\mathcal{N} = 1$ theories. Determining the superfield spectrum is a more complicated problem and an important role is played by constraints on general partial supersymmetry breaking.

Partial supersymmetry breaking has been considered in [47–51]. Following their discussions we briefly summarise how the matter sector of the theory is affected by the breaking. In the $\mathcal{N} = 2$ theory the fields were grouped into multiplets as described in Table 1. Once supersymmetry is broken these multiplets should split up into $\mathcal{N} = 1$ multiplets. The $\mathcal{N} = 2$ gravitational multiplet will need to split into a $\mathcal{N} = 1$ ‘massless’ gravitational multiplet and a massive spin- $\frac{3}{2}$ multiplet [51]

$$(g_{\mu\nu}, \psi_1, \psi_2, A^0) \rightarrow \text{massless } (g_{\mu\nu}, \psi_1) + \text{massive } (\psi_2, A^0, A^1, \chi) \quad (5.1)$$

Here A^1 is a vector field which has to come from one of the vector multiplets and χ is a spin- $\frac{1}{2}$ fermion which come from a hypermultiplet. Moreover, one also needs one Goldstone fermion and two Goldstone bosons to be eaten by the gravitino and the two vector fields respectively which become massive, and these additional Goldstone fields also come from the hypermultiplet sector. Additionally, depending on the details of the theory there will be a certain number of vector and hypermultiplets which also become massive in this process. Integrating out all the massive fields one is left with an $\mathcal{N} = 1$ supergravity theory coupled to vector and chiral multiplets. The scalar fields in an $\mathcal{N} = 1$ theory span a Kähler manifold which has to be a subset of the $\mathcal{N} = 2$ scalar manifold. With the scalar fields of the $\mathcal{N} = 2$ vector multiplets the situation is quite simple as they are already complex coordinates on a (special) Kähler manifold. However, for the hyper-scalars this is not the case, and it is in general non-trivial to find the right combinations which will represent the correct complex coordinates. For simple cases, as we will encounter in this paper, this can be done and one can find explicitly the correct complex combinations which span the $\mathcal{N} = 1$ scalar Kähler manifold.

Before concluding this section we should also mention some subtle issues related to the spontaneous $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking. It has been shown [47–49, 51, 52] that in Minkowski space spontaneous partial supersymmetry breaking can only occur if the symplectic basis in the vector-multiplet sector is such that no prepotential exists. However these results do not apply to the cases we discuss in this paper for the following reasons. First of all, the no-go result above has been obtained for purely electric gaugings of the $\mathcal{N} = 2$ supergravity. Here we will see that we encounter magnetic gaugings as well and going to purely electric gaugings requires to perform some electric-magnetic duality which, in special cases, can take us to a symplectic basis where no prepotential exists. The second argument is that we will encounter the phenomenon of spontaneous partial supersymmetry breaking in AdS space and in such a case it is not clear how to extend the no-go arguments of [47].⁶

⁶We thank Gianguido Dall’Agata for useful discussions on this subject.

5.2 The superfields and Kähler potential

Although the general pattern of partial supersymmetry breaking is constraining it is not enough to determine the superfields in general. The particular difficulty, as explained before, lies in truncating the hypermultiplet spectrum by finding the appropriate Kähler submanifold. However for the special case where we have only the universal hypermultiplet this is possible. We will therefore restrict our general analysis to such a situation anticipating also the fact that the specific example we will study in section 5.4 will be of this type. In order to find models with only one hypermultiplet we will rely on the observation of [53], that six-dimensional manifolds with $SU(3)$ structure for which Ω^+ is exact feature no complex structure moduli and therefore the hypermultiplet sector corresponding to compactifications on such manifolds consists only of the universal hypermultiplet. We therefore restrict ourselves to the case where the torsion classes in (2.11) are restricted to

$$\begin{aligned} \text{Re}(c_1) &= V_2 = S_1 = c_2 = W_2 = A_2 = 0, \\ \text{Im}(c_1) &\neq 0, \end{aligned} \tag{5.2}$$

and we see that under these conditions that the three form Ω^+ is indeed exact.

We further have to determine the gravitino mass matrix for this situation. Using (3.36), (3.14), (3.7) we find that in the particular case considered above, (5.2), the gravitino mass matrix becomes diagonal due to the fact that the internal flux \mathcal{G} has to be closed due to the Bianchi identity

$$\begin{aligned} S_{11} &= \frac{i}{8} \frac{e^{2\phi}}{\sqrt{\mathcal{V}_6}} \int_{\mathcal{M}_7} [dU^+ \wedge U^+ + 2\mathcal{G} \wedge U^+ + 2\lambda], \\ S_{22} &= \frac{i}{8} \frac{e^{2\phi}}{\sqrt{\mathcal{V}_6}} \int_{\mathcal{M}_7} [dU^- \wedge U^- + 2\mathcal{G} \wedge U^- + 2\lambda], \\ S_{12} &= S_{21} = 0. \end{aligned} \tag{5.3}$$

The condition (5.2) appears to be quite strong and we have already come across an example where this is violated in section 4.2. On the other hand we know from ref. [22] that an $\mathcal{N} = 1$ anti-deSitter vacuum, which is required for all the moduli to be stabilised, necessarily means that J is not closed. Hence we always expect at least one of the torsion classes in (5.2) to be non-vanishing. Other than this we must take the condition as a limitation of this paper.

Let us now see how we can identify the surviving degrees of freedom in a spontaneously broken $\mathcal{N} = 2$ theory which comes from a compactification on a manifold which satisfies the requirements above. First of all we know that in order to have partial susy breaking we need at least two Peccei-Quinn isometries of the quaternionic manifold to be gauged such that the corresponding scalar fields become Goldstone bosons which are eaten by the graviphoton and another vector field in the theory. In the model at hand, where we only have one hypermultiplet, we have three such shift symmetries which can be gauged. They correspond to the axion, the dual of the two-form in four dimensions, and the two scalar fields which arise from the expansion of the three-form \hat{c}_3 in the basis of three-forms (α_0, β^0) . In order to gauge one of these last two directions, or a combination thereof, we need that the corresponding combination of the forms α_0 and β^0 is exact. Without loss of

generality we will assume that β^0 is exact. Consistency with equations (3.18) and (3.19) implies then that α_0 is not closed. We therefore see that the scalar field which comes from the expansion in the form β^0 , which we denote $\tilde{\xi}_0$, is a Goldstone boson and will be eaten by one (or a combination) of the vector fields which come from the expansion of C_3 . Then the other Goldstone boson can only be given by the dual of the two form \tilde{B}_2 . The way to see how this direction becomes gauged is obscured by the fact that we are dealing with a two-form rather than directly with a scalar field, but we can note that provided z is not closed, but its derivative is proportional to one of the two forms ω_i , there will appear in the compactified theory a Green-Schwarz interaction, $\tilde{B}_2 \wedge dA$, which upon dualization precisely leads to the desired gauging.⁷ Therefore we learn that the fields which survive the truncation in the $\mathcal{N} = 1$ theory are the dilaton and the second scalar field from the expansion of \hat{c}_3 which we denote by ξ^0 . The final thing which we need to do is to identify the correct complex combination of these two fields which defines the correct coordinate on the corresponding Kähler submanifold. Knowing that the $\mathcal{N} = 2$ gravitino mass matrix becomes the superpotential in the $\mathcal{N} = 1$ theory, which has to be holomorphic in the chiral fields, we are essentially led to the unique possibility

$$U^{0\pm} \equiv \xi^0 \pm ie^{-\phi} \left(\frac{-4iZ^0}{F_0} \right)^{\frac{1}{2}}, \tag{5.4}$$

where the sign \pm is determined by which of the gravitini is massless and we will drop the index unless required for clarity. Z^0 and F_0 are the coefficients of the expansion of Ω in the basis (α_0, β^0) , (3.21), and the quantity $-4iZ^0/F_0$ is a positive real number as in the particular choice of symplectic basis we have made (β_0 is exact) Z^0 is purely imaginary.

To check that this is indeed the correct superfield we should make sure we recover the moduli space metric from the Kähler potential in the gravitino mass. The appropriate kinetic terms in (3.16) read

$$\begin{aligned} S_{kin}^U &= \tag{5.5} \\ &= \int \sqrt{-g} d^4x \left[- \left(\frac{F_0}{-4iZ^0} \right) e^{2\phi} \partial_\mu \left(\xi^0 + ie^{-\phi} \left(\frac{-4iZ^0}{F_0} \right)^{\frac{1}{2}} \right) \partial^\mu \left(\xi^0 - ie^{-\phi} \left(\frac{-4iZ^0}{F_0} \right)^{\frac{1}{2}} \right) \right]. \end{aligned}$$

The gravitino mass in the $\mathcal{N} = 1$ theory is given by the product of the Kähler potential and the superpotential

$$M_{\frac{3}{2}} = e^{\frac{1}{2}K} |W|. \tag{5.6}$$

From this we can use (5.3) to read off the Kähler potential

$$e^{K/2} = \frac{e^{2\phi}}{\sqrt{8\mathcal{V}_6}}, \tag{5.7}$$

It is then easily shown that indeed the superfield and Kähler potential satisfy

$$\partial_{U^0} \partial_{\bar{U}^0} \ln \left[\frac{e^{4\phi}}{8\mathcal{V}_6} \right] = - \left(\frac{F_0}{-4iZ^0} \right) e^{2\phi}. \tag{5.8}$$

⁷The issue of the dualization is further obstructed by the fact that B will be massive. This, as explained at the end of section 3.3, is triggered by the non-closure of the one form z , which leads to mass term for the two-form field \tilde{B}_2 of the type $\int dz \wedge *dz$.

Hence we have identified the correct superfield in the truncated spectrum. Determining the superfields arising from the $\mathcal{N} = 2$ vector multiplets is a much easier task as they are just the natural pairing found in (3.17)

$$t^i \equiv b^i - iw^i, \tag{5.9}$$

where the index i now runs over the lower mass fields.

5.3 The superpotential

The superpotential for the $\mathcal{N} = 1$ theory can be read off from the gravitino mass to be

$$W = \frac{i}{\sqrt{8}} \left\{ \int_{\mathcal{M}_7} [dU^\pm \wedge U^\pm] + \mathcal{G} \wedge U^\pm + 2\lambda \right\}, \tag{5.10}$$

where again the \pm sign is fixed by the lower mass state. From this expression for the superpotential we can see that we should generically expect a constant term λ , linear terms in U , quadratic terms t^2 , U^2 as well as mixed terms tU . These type of potentials will, in general, stabilise all the moduli and we will see such an example in the next section.

It is instructive to note that finding a supersymmetric solution for this superpotential automatically solves the equations which are required for a solution of the full $\mathcal{N} = 2$ theory to preserve some supersymmetry. Therefore, for such a solution, it would be enough to show, using the mass matrix (5.3), that a mass gap between the two gravitini forms in order to prove that partial supersymmetry breaking does indeed occur.

5.4 The Coset $SU(3) \times U(1)/U(1) \times U(1)$

In this section we will go through an explicit example of a manifold that preserves $\mathcal{N} = 1$ supersymmetry in the vacuum. The manifold we will be considering is the coset $SU(3) \times U(1)/U(1) \times U(1)$ and for simplicity we shall turn off the four-form flux $\mathcal{G} = 0$. Details of the structure of the coset can be found in the appendix and in this section we summarise the relevant parts. The coset is specified by three integers p, q , and r that determine the embeddings of the $U(1) \times U(1)$ in $SU(3) \times U(1)$, where the integers satisfy

$$0 \leq 3p \leq q, \tag{5.11}$$

with all other choices corresponding to different parameterisations of the $SU(3)$. As with the previous coset example we can use the coset symmetries to derive the invariant $SU(3)$ structure forms and the metric. The metric is given by

$$g = \begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d \end{pmatrix}, \tag{5.12}$$

where the parameters a, b, c, d are all real. We can write the invariant forms as

$$\begin{aligned} V &= \sqrt{d}z, \\ J &= a\omega_1 + b\omega_2 + c\omega_3, \\ \Omega &= \sqrt{abc}(i\alpha_0 - 4\beta^0). \end{aligned} \tag{5.13}$$

This basis can be shown to satisfy the following differential relations

$$\begin{aligned} dz &= m^i\omega_i, \\ d\omega_i &= e_i\beta^0, \quad d\tilde{\omega}^i = 0, \\ d\alpha_0 &= e_i\tilde{\omega}^i, \quad d\beta^0 = 0, \end{aligned} \tag{5.14}$$

where we have introduced two vectors $e_i = (2, 2, 2)$, and $m^i = (\alpha, -\beta, \gamma)$, $i = 1, 2, 3$ which encode the information about the metric fluxes. The quantities α , β and γ are not independent, but satisfy $\alpha - \beta + \gamma = 0$ and in terms of the integers p and q have the expressions

$$\begin{aligned} \alpha &\equiv \frac{q}{\sqrt{3p^2 + q^2}}, \\ \beta &\equiv \frac{3p + q}{2\sqrt{3p^2 + q^2}}, \\ \gamma &\equiv \frac{3p - q}{2\sqrt{3p^2 + q^2}}. \end{aligned} \tag{5.15}$$

This ends our summary of the relevant features of the coset. We see that this manifold indeed has the required torsion classes (5.2) and, as expected, has no complex structure moduli and three Kähler moduli.

5.4.1 $\mathcal{N} = 1$ minimum

As explained in [54], M-theory compactifications on the coset manifold presented above are expected to preserve $\mathcal{N} = 1$ supersymmetry in the vacuum. Therefore we can use the machinery developed at the beginning of this section and derive the $\mathcal{N} = 1$ theory in the vacuum. We will also turn off the four-form flux \mathcal{G} and so, using equations (5.7) and (5.10) we find the superpotential and Kähler potential to be

$$W = \frac{1}{\sqrt{8}} [4U^0 (t^1 + t^2 + t^3) + 2\alpha t^2 t^3 - 2\beta t^1 t^3 + 2\gamma t^1 t^2 + 2\lambda], \tag{5.16}$$

$$K = -4\ln [-i (U^0 - \bar{U}^0)] - \ln [-i (t^1 - \bar{t}^1) (t^2 - \bar{t}^2) (t^3 - \bar{t}^3)] + \text{const.} \tag{5.17}$$

where the superfields t^i were defined in (5.9) while for U^0 we have

$$U^{0\pm} = \xi^0 \pm i e^{-\phi}, \tag{5.18}$$

as (5.13) gives $-4iZ^0/F_0 = 1$. We can look for supersymmetric vacua to this action by solving the F-term equations. For convenience we restrict to the family of cosets with $p = 0$ though the results can be reproduced for more general choices of embeddings. We find the

solution to the F-term equations

$$\frac{t^1}{2} = t^2 = t^3 = U^0 = -i\sqrt{\frac{\lambda}{3}}. \tag{5.19}$$

At this point we can go back to check which of the gravitini is more massive. Inserting the solution (5.19) into the expression of the mass matrix (5.3) we obtain

$$S_{11} > S_{22}, \tag{5.20}$$

which means ψ^2 is the lighter gravitino and the one that should be kept in the truncated theory. This gravitino is physically massless as expected. This also fixes the \pm sign ambiguity in the superfield and superpotential so that we have $U^0 \equiv U^{0-}$. Finally we note that as this solution is a supersymmetric solution of the truncated $\mathcal{N} = 1$ theory and that according to (5.20) the gravitino masses are not degenerate we indeed have encountered the phenomenon of partial super symmetry breaking.

5.4.2 The structure in the vacuum

It is informative to look at the form of the G structure of the coset in the vacuum in terms of the G_2 structures. The two G_2 forms (2.13) satisfy the vacuum differential and algebraic relations

$$\begin{aligned} d\varphi^\pm &= \sqrt{2} \left(\frac{\lambda}{3}\right)^{\frac{3}{4}} [-8\beta^0 \wedge z \pm 2\omega_1 \wedge \omega_2 + (\pm 2 + 1)\omega_2 \wedge \omega_3 \pm 2\omega_1 \wedge \omega_3], \\ \frac{2}{3}f \star \varphi^\pm &= \sqrt{2} \left(\frac{\lambda}{3}\right)^{\frac{3}{4}} [\pm 8\beta^0 \wedge z - 2\omega_1 \wedge \omega_2 - \omega_2 \wedge \omega_3 - 2\omega_1 \wedge \omega_3]. \end{aligned} \tag{5.21}$$

It is clear to see that only φ^- is weak- G_2 , and this is indeed the G_2 structure that features in the superpotential and is associated with the lower mass gravitino. This shows an explicit mass gap appearing between the two G_2 structures which is the same mass gap that corresponds to the partial supersymmetry breaking which we have used to write an effective $\mathcal{N} = 1$ theory. Hence we have shown an example of the idea of an effective G structure where we could have arrived at this truncated $\mathcal{N} = 1$ theory through a G_2 structure compactification even though the manifold actually has $SU(3)$ structure. Finally we should note that we could have used the condition that the manifold should be weak- G_2 in the vacuum to solve for the values of the moduli in the vacuum as we did in section 4.2.1 instead of solving the F-term equations.

6. Conclusions

In this paper we studied compactifications of M-theory on manifolds with $SU(3)$ structure. We showed that these compactifications, under certain assumptions regarding the expansion basis discussed in section 3.2.1, can be cast into a form much like type IIA compactifications on six-dimensional manifolds with $SU(3)$ structure. The classical potential

for the fields in four-dimensions differed however from the IIA case and we have proved in two explicit examples that one can find vacua which fix all the moduli without the need of non-perturbative effects.

We have also shown that depending on the different torsion classes which can be turned on for such manifolds one can arrange to preserve either $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetry. We have also argued that in the case of the $\mathcal{N} = 1$ solution one encounters the phenomenon of partial supersymmetry breaking. This arises due to the fact that the two spinors which define the $SU(3)$ structure satisfy different differential relations – or in other words, they are eigenfunctions of the Dirac operator corresponding to different eigenvalues – leading in this way to different masses for the corresponding gravitini. In such a case we have seen that effectively one can ignore from the beginning one of the spinors which make up the $SU(3)$ structure leading in this way to a G_2 -like compactification.

There are many interesting directions that can be followed from this paper. It would be interesting to consider manifolds that are more general than the restriction (5.2) and in particular the case where both the c_1 and c_2 torsion classes are non-vanishing should lead to a theory with a vacuum that preserves $\mathcal{N} = 1$ supersymmetry and has a stable vacuum where the axions are stabilised at non-zero values. This would correspond to the unwarped solution with non-vanishing exact internal flux found in [22].

We have not touched on the subject of realistic particle content in this paper one reason being that one can not possibly achieve a viable spectrum of particles in M-theory compactifications by considering smooth manifolds as we do in this paper. However, in the effort to construct four-dimensional theories which contain chiral matter and gauge fields from M-theory compactifications (for recent developments see [55]), considering seven-dimensional manifolds with $SU(3)$ structure should be very interesting because, as shown in this paper one can easily fix all the bulk moduli. This could be supplemented by turning on torsion classes that would lead to off-diagonal terms in the mass matrix that can be interpreted as D-terms in the effective $\mathcal{N} = 1$ theory thereby breaking supersymmetry spontaneously.

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A. Conventions

In this appendix we outline the conventions used throughout this paper. The index ranges are

$$M, N, P, Q, R, S, T, U, V, W = 0, \dots, 10,$$

$$\begin{aligned}
a, b, m, n, p, q, r, s, t &= 0, \dots, 6, \\
\mu, \nu, \rho &= 0, \dots, 3 \\
i, j, k &= 1, \dots, \text{Number of two - forms in the basis,} \\
A, B &= 1, \dots, \text{Number of three - forms in the basis,} \\
\alpha, \beta &= 1, 2.
\end{aligned}
\tag{A.1}$$

We worked with a mostly plus metric signature

$$\hat{\eta}_{11} = (-1, +1, +1, \dots), \tag{A.2}$$

where generally $\hat{}$ denotes eleven-dimensional quantities. The $\hat{\epsilon}$ tensor density is defined as

$$\hat{\epsilon}_{0123\dots} = +1, \tag{A.3}$$

and we define the inner product between forms as

$$(\omega_p \lrcorner \nu_q)_{\mu_{p+1}\dots\mu_q} \equiv \frac{1}{p!} (\omega_p)^{\mu_1\dots\mu_p} (\nu_q)_{\mu_1\dots\mu_p\mu_{p+1}\dots\mu_q}. \tag{A.4}$$

The eleven-dimensional spinor conventions are such that the charge conjugation operator is given by $\hat{\Gamma}_0$

$$\bar{\hat{\Psi}} = \hat{\Psi}^\dagger \hat{\Gamma}_0. \tag{A.5}$$

We decompose the eleven-dimensional gamma matrices as

$$\begin{aligned}
\hat{\Gamma}_\mu &= \gamma_\mu \otimes 1, \\
\hat{\Gamma}_m &= \gamma_5 \otimes \gamma_m,
\end{aligned}
\tag{A.6}$$

with γ_m imaginary and γ_μ real and

$$\begin{aligned}
-i\gamma_{0123} &= \gamma_5, \\
\gamma_{01\dots 6} &= -i.
\end{aligned}
\tag{A.7}$$

B. Ricci scalar reduction

In this appendix we reduce the eleven-dimensional Ricci scalar using the metric Ansatz (3.2). Before we begin the calculation we should comment on the kind of variations we consider here. In general, seven-dimensional manifolds with $SU(3)$ structure can have isometries that produce gauge fields in the effective lower dimensional theory. For the moment we are not interested in such metric variations and only treat the scalar modes which appear from the fluctuations of the metric on the internal manifold. Moreover we are only interested in the lightest modes in the Kaluza–Klein tower. Thus we consider a metric, including the fluctuations, of the following form

$$\begin{aligned}
ds_{11}^2 &= \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + \bar{g}_{mn}(x, y) dy^m dy^n \\
&= \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + [\bar{g}_{mn}^0(y) + \bar{h}_{mn}(x, y)] dy^m dy^n.
\end{aligned}
\tag{B.1}$$

Direct computation of the 11d Ricci scalar gives

$$\begin{aligned}
 & \int \sqrt{-g_{11}} d^{11} X \frac{1}{2} R_{11} \\
 &= \int \sqrt{-g_{11}} d^{11} X \frac{1}{2} \left[\bar{R}_4 + \bar{R}_7 - \bar{g}^{mn} \bar{\square}_4 \bar{g}_{mn} + \left(\frac{3}{4} \bar{g}^{mp} \bar{g}^{nq} - \frac{1}{4} \bar{g}^{mn} \bar{g}^{pq} \right) (\partial \bar{g}_{mn}) (\partial \bar{g}_{pq}) \right] \\
 &= \int \sqrt{-\bar{g}_4} d^4 x \int \sqrt{\bar{g}} d^7 y \frac{1}{2} \left[\bar{R}_4 + \bar{R}_7 - \frac{1}{4} (\bar{g}^{mp} \bar{g}^{nq} - \bar{g}^{mn} \bar{g}^{pq}) (\partial \bar{g}_{mn}) (\partial \bar{g}_{pq}) \right],
 \end{aligned}$$

where in the last equation we have performed a partial integration with respect to the four-dimensional integral. At this point we want to replace the metric variations with variations of the structure forms. Although eventually we wish to parameterise the variations in terms of the $SU(3)$ structure forms at this point it is easier to work with the $G2$ forms. Using equation (3.5) we arrive at

$$\int \sqrt{-g_{11}} d^{11} X R_{11} = \int \sqrt{-\bar{g}_4} d^4 x \int \sqrt{\bar{g}} \left[\bar{R}_4 + \bar{R}_7 - \frac{1}{12} (\partial \bar{\varphi})_{mnp} (\partial \bar{\varphi})^{mnp} + \frac{3}{2} \frac{(\partial \bar{\mathcal{V}})^2}{\bar{\mathcal{V}}^2} \right], \quad (\text{B.2})$$

where to reach this we used the $G2$ identities

$$\begin{aligned}
 \varphi_m^{pq} \varphi_{ab}^m &= (\star \varphi)_{ab}^{pq} + 2 \delta_{ab}^{pq}, \\
 9 (\star \varphi)_{[ab}^{[pq} \delta_{n]}^m] &= (\star \varphi)^{pqmt} (\star \varphi)_{abnt} + \varphi^{pqm} \varphi_{abn} - 6 \delta_{abn}^{pqm}, \\
 \varphi \lrcorner \delta \varphi &= 3 \mathcal{V}^{-1} \delta \mathcal{V},
 \end{aligned} \quad (\text{B.3})$$

and the fact that only the symmetric part of $\varphi_m^{pq} \delta \varphi_{npq}$ contributes to the gauge independent metric variations. Here $\bar{\mathcal{V}}$ is the volume of the internal manifold as measured with the metric \bar{g}_{mn} which thus contains the metric fluctuations. Note that because we only consider the lowest KK states, \bar{R}_4 is independent of the internal coordinates and thus its integration produces a factor of the seven-dimensional volume $\bar{\mathcal{V}}$. In order to put the four-dimensional action in the standard form we further need to rescale the four dimensional metric as

$$\bar{g}_{\mu\nu} = \frac{1}{\bar{\mathcal{V}}} g_{\mu\nu}. \quad (\text{B.4})$$

Apart from normalising the Einstein-Hilbert term correctly this rescaling will also produce a term which precisely cancels the last term of (B.2). Thus the final form of the compactified eleven-dimensional Ricci scalar takes the form

$$\int \sqrt{-g_{11}} d^{11} X R_{11} = \int \sqrt{-g_4} d^4 x \left[R_4 + \int \sqrt{\bar{g}} (\bar{R}_7 - \frac{1}{12} (\partial \bar{\varphi})_{mnp} (\partial \bar{\varphi})^{mnp}) \right]. \quad (\text{B.5})$$

At this stage we can move back to using the $SU(3)$ forms using the translation equation (2.13). We also move to the string frame by rescaling the internal metric

$$\bar{g}_{mn} = e^{-\frac{2}{3} \hat{\phi}} g_{mn}, \quad (\text{B.6})$$

where the dilaton is defined as in equation (3.6). Defining the $SU(3)$ structure forms with respect to the metric g_{mn} the decomposition (2.13) becomes

$$\bar{\varphi}^\pm = e^{-\hat{\phi}} (\pm \Omega^- - J \wedge V). \quad (\text{B.7})$$

Before identifying the correct degrees of freedom in four dimensions, as discussed in section 3.2 we need to take out the Kähler moduli dependence from Ω and we do this by defining a 'six-dimensional' volume \mathcal{V}_6 and the true 'holomorphic' three-form Ω^{cs} as in equations (3.7) and (3.10). With these definitions we have

$$\partial\bar{\varphi}^\pm = e^{-\phi} \left(\pm (\partial\phi) e^{\frac{1}{2}K_{cs}} \Omega_{cs}^- \pm \partial \left(e^{\frac{1}{2}K_{cs}} \Omega_{cs}^- \right) - \frac{1}{\sqrt{\mathcal{V}_6}} \partial J \wedge V \right), \quad (\text{B.8})$$

where it can be easily checked that

$$\left(\partial \left(e^{\frac{1}{2}K_{cs}} \Omega_{cs}^- \right) \right)_{mnp} \left(e^{\frac{1}{2}K_{cs}} \Omega_{cs}^- \right)^{mnp} = 0, \quad (\text{B.9})$$

and so when we square the expression (B.8) there is no mixing between the various terms. Inserting (B.8) into (B.5) we arrive at the final expression (3.13).

C. The gravitini mass matrix

In this appendix we will derive the four-dimensional gravitini mass matrix through dimensional reduction of the appropriate terms in the eleven-dimensional action. We wish to work in terms of the $SU(3)$ structure quantities as defined in section 2.2 and so we begin by writing the eleven-dimensional gravitino ansatz (3.32) in terms of the four-dimensional chiral gravitini (3.32) and the complex internal spinors (2.3)

$$\hat{\Psi}_\mu = \mathcal{V}^{-\frac{1}{4}} \left[(\psi_{+\mu}^1 + \psi_{-\mu}^1) \otimes (\eta_+ + \eta_-) - i (\psi_{+\mu}^2 + \psi_{-\mu}^2) \otimes (\eta_+ - \eta_-) \right]. \quad (\text{C.1})$$

We now go through each term in (3.1) that will contribute to the four-dimensional mass matrix.

The kinetic term We begin with the eleven-dimensional kinetic term which will produce a mass term in four dimensions for the particular index range choices

$$\mathcal{L}_1 = -\frac{1}{2} \bar{\Psi}_\mu \hat{\Gamma}^{\mu\nu} \hat{D}_n \Psi_\nu. \quad (\text{C.2})$$

This term is only non-vanishing when the internal spinors are not covariantly constant and so will correspond to the potential induced by the torsion on the manifold. To calculate this more precisely we use the relation for the covariant derivative acting on the spinors

$$D_m \eta_\pm = \frac{1}{4} \kappa_{mnp} \gamma^{np} \eta_\pm, \quad (\text{C.3})$$

where κ_{mnp} is the contorsion on the internal manifold which is anti-symmetric in its last two indices. Inserting (C.1) into (C.2) and using (C.3) to evaluate the derivative on the spinors as well as (2.4) to replace the spinor bi-linears with the $SU(3)$ forms we arrive at

$$\begin{aligned} \mathcal{L}_1 = & -\frac{1}{2\mathcal{V}^{\frac{1}{2}}} \left\{ \bar{\psi}_{+\mu}^1 \gamma^{\mu\nu} \psi_{-\nu}^1 \left[\frac{i}{2} \kappa_{[mnp]} (J \wedge V)^{mnp} - \frac{i}{2} \kappa_{[mnp]} \Omega^{-mnp} \right] \right. \\ & \left. + \bar{\psi}_{+\mu}^2 \gamma^{\mu\nu} \psi_{-\nu}^2 \left[\frac{i}{2} \kappa_{[mnp]} (J \wedge V)^{mnp} + \frac{i}{2} \kappa_{[mnp]} \Omega^{-mnp} \right] \right\} \quad (\text{C.4}) \end{aligned}$$

$$\begin{aligned}
& + \bar{\psi}^1_{+\mu} \gamma^{\mu\nu} \psi^2_{-\nu} \left[-i\kappa_{m[np]} V^{[n} \delta^p]m} - \frac{i}{2} \kappa_{[mnp]} \Omega^{+mnp} \right] \\
& + \bar{\psi}^2_{+\mu} \gamma^{\mu\nu} \psi^1_{-\nu} \left[i\kappa_{m[np]} V^{[n} \delta^p]m} - \frac{i}{2} \kappa_{[mnp]} \Omega^{+mnp} \right] + \text{c.c.} \left. \right\} .
\end{aligned}$$

Now using the identity

$$\bar{\psi}^2_{+\mu} \gamma^{\mu\nu} \psi^1_{-\nu} = \bar{\psi}^1_{+\mu} \gamma^{\mu\nu} \psi^2_{-\nu} , \tag{C.5}$$

we can see that actually the first terms in the third and fourth lines cancel. This can be reasoned from the fact that the mass matrix should be symmetric. Using (C.3) we can operate on the spinor bi-linears (2.4) and derive the following useful relations

$$\begin{aligned}
(dV)_{mn} &= 2\kappa_{[mn]p} V^p , \\
(dJ)_{mnp} &= 6\kappa_{[mn}{}^r J_{r|p]} , \\
(d\Omega)_{mnpq} &= 12\kappa_{[mn}{}^r \Omega_{r|pq]} .
\end{aligned} \tag{C.6}$$

With this (C.6) we can eliminate the contorsion from (C.5) in favour of differential relations of the structure forms and we obtain

$$\begin{aligned}
\mathcal{L}_1 &= -\frac{1}{2\mathcal{V}^{\frac{1}{2}}} \left\{ \bar{\psi}^1_{+\mu} \gamma^{\mu\nu} \psi^1_{-\nu} \right. \\
&\quad \times \left[\frac{i}{4} (dV)_{mn} J^{mn} + \frac{i}{96} (d\Omega^-)_{mnpq} (\star\Omega^-)^{mnpq} + \frac{i}{12} (dJ)_{mnp} (\Omega^+)^{mnp} \right] \\
&\quad + \bar{\psi}^2_{+\mu} \gamma^{\mu\nu} \psi^2_{-\nu} \\
&\quad \times \left[\frac{i}{4} (dV)_{mn} J^{mn} + \frac{i}{96} (d\Omega^-)_{mnpq} (\star\Omega^-)^{mnpq} - \frac{i}{12} (dJ)_{mnp} (\Omega^+)^{mnp} \right] \\
&\quad + \bar{\psi}^1_{+\mu} \gamma^{\mu\nu} \psi^2_{-\nu} \left[-\frac{i}{12} (dJ)_{mnp} (\Omega^-)^{mnp} \right] \\
&\quad \left. + \bar{\psi}^2_{+\mu} \gamma^{\mu\nu} \psi^1_{-\nu} \left[-\frac{i}{12} (dJ)_{mnp} (\Omega^-)^{mnp} \right] + \text{c.c.} \right\} . \tag{C.7}
\end{aligned}$$

This concludes the reduction of the kinetic term and we now move on to the flux terms.

The flux terms We begin by reducing the term

$$\mathcal{L}_2 = -\frac{1}{16} \bar{\Psi}^\mu \hat{\Gamma}^{\rho\sigma} \Psi^\nu F_{\mu\rho\sigma\nu} . \tag{C.8}$$

This term arises from the purely external Freund-Rubin flux which we write as in (3.27) and (3.30). Then substituting (C.1) into (C.8) and after some gamma matrix algebra we arrive at

$$\mathcal{L}_2 = [i\bar{\psi}^1_{+\mu} \gamma^{\mu\nu} \psi^1_{-\nu} + i\bar{\psi}^2_{+\mu} \gamma^{\mu\nu} \psi^2_{-\nu} + \text{c.c.}] \left[\frac{1}{4\mathcal{V}^{\frac{3}{2}}} \left(\lambda + \frac{1}{2} \int \hat{c}_3 \wedge F \right) \right] . \tag{C.9}$$

The second flux term reads

$$\mathcal{L}_3 = -\frac{3}{4(12)^2} \bar{\Psi}_\mu \hat{\Gamma}^{\mu\nu l m n p} \Psi_\nu F_{lmnp} . \tag{C.10}$$

This is the term from the purely internal flux. Again the reduction is simple and gives

$$\begin{aligned} \mathcal{L}_3 = \frac{1}{4(12)^2 \mathcal{V}^{\frac{1}{2}}} & \left\{ \bar{\psi}_{+\mu}^1 \gamma^{\mu\nu} \psi_{-\nu}^1 \left[F^{lmnp} (J \wedge V - \Omega^-)^{rst} \hat{\epsilon}_{lmnprst} \right] \right. \\ & + \bar{\psi}_{+\mu}^2 \gamma^{\mu\nu} \psi_{-\nu}^2 \left[F^{lmnp} (J \wedge V + \Omega^-)^{rst} \hat{\epsilon}_{lmnprst} \right] \\ & + \bar{\psi}_{+\mu}^1 \gamma^{\mu\nu} \psi_{-\nu}^2 \left[-F^{lmnp} (\Omega^+)^{rst} \hat{\epsilon}_{lmnprst} \right] \\ & \left. + \bar{\psi}_{+\mu}^2 \gamma^{\mu\nu} \psi_{-\nu}^1 \left[-F^{lmnp} (\Omega^+)^{rst} \hat{\epsilon}_{lmnprst} \right] + \text{c.c.} \right\}. \end{aligned} \quad (\text{C.11})$$

Finally we recall that the purely internal flux has a contribution from the the background flux \mathcal{G} , and one which is due to the torsion of the internal manifold $d\hat{c}_3$, which combine into

$$F_{lmnp} = \mathcal{G}_{lmnp} + (d\hat{c}_3)_{lmnp}. \quad (\text{C.12})$$

After performing the Weyl rescalings (3.12), the contributions computed above, (C.7), (C.9), and (C.12) yield the following mass terms for the gravitino in four dimensions

$$\tilde{S}_{\text{mass}} = \int_{\mathcal{M}_{11}} \sqrt{-g} [\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3] = \int_{\mathcal{M}_4} \sqrt{-g} \left[S_{\alpha\beta} \bar{\psi}_{+\mu}^\alpha \gamma^{\mu\nu} \psi_{-\nu}^\beta + \text{c.c.} \right], \quad (\text{C.13})$$

where

$$\begin{aligned} S_{11} &= -\frac{ie^{\frac{3}{2}\hat{\phi}}}{8\mathcal{V}^{\frac{3}{2}}} \left\{ \int_{\mathcal{M}_7} \left[d\Omega^- \wedge \Omega^- + dV \wedge V \wedge J \wedge J + 2dJ \wedge \Omega^- \wedge V \right. \right. \\ & \quad \left. \left. - 2\mathcal{G} \wedge (\hat{c}_3 + i(\Omega^- - J \wedge V)) - d\hat{c}_3 \wedge \hat{c}_3 \right. \right. \\ & \quad \left. \left. - 2id\hat{c}_3 \wedge (\Omega^- - J \wedge V) \right] - 2\lambda \right\}, \\ S_{22} &= -\frac{ie^{\frac{3}{2}\hat{\phi}}}{8\mathcal{V}^{\frac{3}{2}}} \left\{ \int_{\mathcal{M}_7} \left[d\Omega^- \wedge \Omega^- + dV \wedge V \wedge J \wedge J - 2dJ \wedge \Omega^- \wedge V \right. \right. \\ & \quad \left. \left. - 2\mathcal{G} \wedge (\hat{c}_3 + i(-\Omega^- - J \wedge V)) - d\hat{c}_3 \wedge \hat{c}_3 \right. \right. \\ & \quad \left. \left. - 2id\hat{c}_3 \wedge (-\Omega^- - J \wedge V) \right] - 2\lambda \right\}, \\ S_{12} = S_{21} &= -\frac{ie^{\frac{3}{2}\hat{\phi}}}{8\mathcal{V}^{\frac{3}{2}}} \int_{\mathcal{M}_7} \left[2dJ \wedge \Omega^+ \wedge V - 2ie^{\hat{\phi}} \mathcal{G} \wedge \Omega^+ - 2ie^{\hat{\phi}} d\hat{c}_3 \wedge \Omega^+ \right]. \end{aligned} \quad (\text{C.14})$$

This action can be written in the form (3.36) using (3.37).

D. Coset manifolds

In this appendix we wish to briefly describe the procedure through which we can derive explicit information on the coset such as the metric, the G structure forms and the basis forms and their differential relations.

Consider a compact group G with some subgroup H then we can decompose the Lie algebra as $\mathfrak{g} = \mathfrak{H} \oplus \mathfrak{K}$. So the Lie manifold \mathcal{M}_g is a fibration of the Lie manifold \mathcal{M}_H over the base \mathcal{M}_K . The base manifold \mathcal{M}_K is the coset manifold $\frac{G}{H}$. We now follow the discussion in [56] and construct a set of Lie valued one-forms from elements on the fibre

L_y at a point y on the coset manifold, which we then expand in terms of the generators of the groups H and K

$$\Theta \equiv L_y^{-1} dL_y \equiv \sigma^a H_a + e^i K_i, \tag{D.1}$$

where the indices run over the number of generators of the subgroup. The forms e^i will form the basis forms on the coset manifold and using

$$d\Theta = dL^{-1} \wedge dL = -L^{-1} dL \wedge L^{-1} dL = -\Theta \wedge \Theta, \tag{D.2}$$

gives that the basis forms satisfy the differential relations

$$\begin{aligned} d\sigma^a &= -\frac{1}{2} f_{bc}^a \sigma^b \wedge \sigma^c - \frac{1}{2} f_{ij}^a e^i \wedge e^j, \\ de^i &= -\frac{1}{2} f_{jk}^i e^j \wedge e^k - f_{aj}^i \sigma^a \wedge e^j, \end{aligned} \tag{D.3}$$

where f are the structure constants of the group G . These expressions allow us to calculate the differential relations on the coset. The useful property of the coset is that requiring G -invariance

$$gL_y = L_{y'} h, \tag{D.4}$$

where $g \in G$ and $h \in H$, we recover the transformation rules for a basis form on the coset

$$e^i(y') K_i = e^i(y) h K_i h^{-1}, \tag{D.5}$$

which means that requiring homogeneity of the basis forms general n -tensor on the coset

$$g = g_{i_1 \dots i_n} e^{i_1} \otimes \dots \otimes e^{i_n}, \tag{D.6}$$

should satisfy the relation

$$f_{ai_1}^j g_{ji_2 \dots i_n} + \dots + f_{ai_n}^j g_{i_1 \dots j} = 0, \quad \forall a, \tag{D.7}$$

and should have constant co-efficients $g_{i_1 \dots i_n}$. This is the expression that restricts the possible forms that respect the coset symmetries which we can use to solve for the most general one-, two-, or three-forms on the coset and also the metric. Having quickly derived the relevant expressions (D.3) and (D.7) we can move on to consider the particular examples used in this paper. One immediate conclusion we can draw is that scalar functions must be constant. This is the general result that cosets can not support warping.

D.1 $SO(5)/SO(3)_{A+B}$

The group $SO(5)$ has two commuting $SO(3)$ subgroups. Hence there are a number of ways to mod out the $SO(3)$ and the index $A+B$ refers to the case where the subgroup H is taken to be a linear combination of the two $SO(3)$ s. Then by calculating the structure constants and imposing (D.7) we find that the most general symmetric two tensor on the coset, which we interpret as the metric, must take the form

$$g = a(e^1 \otimes e^1 + e^2 \otimes e^2 + e^3 \otimes e^3) + b e^4 \otimes e^4 + c(e^5 \otimes e^5 + e^6 \otimes e^6 + e^7 \otimes e^7)$$

$$+2d(e^{(1} \otimes e^5) + e^{(2} \otimes e^6) + e^{(3} \otimes e^7)), \quad (\text{D.8})$$

where all the parameters are real. Similarly, the most general one-, two-, and three-forms are

$$\begin{aligned} \Psi_1 &= \zeta_1 e^4, \\ \Psi_2 &= \zeta_2 (e^{15} + e^{26} + e^{37}), \\ \Psi_3 &= \zeta_3 e^{123} + \zeta_4 (e^{127} - e^{136} + e^{235}) + \zeta_5 (e^{145} + e^{246} + e^{347}) \\ &\quad + \zeta_7 e^{567} + \zeta_6 (e^{167} - e^{257} + e^{356}), \end{aligned} \quad (\text{D.9})$$

where all the parameters can be complex. The structure forms V , J and Ω must fall within the restrictions of (D.9) and they can be uniquely determined by imposing the algebraic $SU(3)$ structure relations on the forms in (2.6). This leads to equations relating the complex parameters to the real metric moduli, if we identify Ψ_1 with V , Ψ_2 with J , Ψ_3 with Ω .

$$\begin{aligned} \zeta_1 &= \sqrt{b}, \\ \zeta_2 &= (ac - d^2)^{\frac{1}{2}}, \\ \zeta_3 &= \frac{\zeta_6}{a^2} \left(d + i(ac - d^2)^{\frac{1}{2}} \right)^2, \\ \zeta_4 &= \frac{\zeta_6 a}{\left(d + i(ac - d^2)^{\frac{1}{2}} \right)}, \\ \zeta_5 &= 0, \\ \zeta_6 &= \frac{2(ac - d^2)^{\frac{1}{2}} a \sqrt{c}}{a + ic}, \\ \zeta_7 &= \frac{\zeta_6 c}{\left(d - i(ac - d^2)^{\frac{1}{2}} \right)}. \end{aligned} \quad (\text{D.10})$$

Equations (D.10) give the form of V , J and Ω and we see that the natural basis of forms on the manifold is

$$\begin{aligned} z &\equiv e^4, \\ \omega &\equiv (e^{15} + e^{26} + e^{37}), \\ \alpha_0 &\equiv e^{123} \quad \beta^0 \equiv e^{567}, \\ \alpha_1 &\equiv (e^{127} - e^{136} + e^{235}) \quad \beta^1 \equiv (e^{167} - e^{257} + e^{356}), \end{aligned} \quad (\text{D.11})$$

in terms of which we can write the forms as given in equation (4.20). The differential relations on the coset basis forms can be calculated using (D.3) and are given by

$$\begin{aligned} d\sigma^1 &= -\sigma^{23} - e^{23} - e^{67}, \\ d\sigma^2 &= \sigma^{13} + e^{13} + e^{57}, \\ d\sigma^3 &= -\sigma^{12} - e^{12} - e^{56}, \end{aligned}$$

$$\begin{aligned}
de^1 &= -\sigma^2 e^3 + \sigma^3 e^2 + e^{45}, \\
de^2 &= \sigma^1 e^3 - \sigma^3 e^1 + e^{46}, \\
de^3 &= -\sigma^1 e^2 + \sigma^2 e^1 + e^{47}, \\
de^4 &= -e^{15} - e^{26} - e^{37}, \\
de^5 &= -\sigma^2 e^7 + \sigma^3 e^6 + e^{14}, \\
de^6 &= \sigma^1 e^7 - \sigma^3 e^5 + e^{24}, \\
de^7 &= -\sigma^1 e^6 + \sigma^2 e^5 + e^{34},
\end{aligned}
\tag{D.12}$$

From these expressions it is easy to calculate the basis form differential relations (4.21).

D.2 $SU(3) \times U(1)/U(1) \times U(1)$

This coset was first studied in [54]. In this case we have $G = SU(3) \times U(1)$. Now $U(1) \times U(1) \subset SU(3)$ so once we modded out by the $U(1) \times U(1)$ we will be left with a single $U(1)$ that is in general a linear combination of the three $U(1)$ s in G which we parameterise by three integers p, q and r ⁸. We can repeat the analysis in the previous section and we find

$$\begin{aligned}
g &= a(e^1 \otimes e^1 + e^2 \otimes e^2) + b(e^3 \otimes e^3 + e^4 \otimes e^4) + c(e^5 \otimes e^5 + e^6 \otimes e^6) + de^7 \otimes e^7, \\
\Psi_1 &= \zeta_1 e^7, \\
\Psi_2 &= \zeta_2 e^{12} + \zeta_3 e^{34} + \zeta_4 e^{56}, \\
\Psi_3 &= \zeta_5 (e^{135} + e^{146} - e^{236} + e^{245}) + \zeta_6 (e^{136} - e^{145} + e^{235} + e^{246}),
\end{aligned}
\tag{D.13}$$

Imposing the $SU(3)$ relations we arrive at equation (5.13) where the basis forms explicitly read

$$\begin{aligned}
z &\equiv e^7, \\
\omega_1 &\equiv -e^{12} \quad \omega_2 \equiv e^{34} \quad \omega_3 \equiv -e^{56}, \\
\tilde{\omega}^1 &\equiv -e^{3456} \quad \tilde{\omega}^2 \equiv e^{1256} \quad \tilde{\omega}^3 \equiv -e^{1234}, \\
\alpha_0 &\equiv (-e^{136} + e^{145} - e^{235} - e^{246}) \quad \beta^0 \equiv -\frac{1}{4} (e^{135} + e^{146} - e^{236} + e^{245}),
\end{aligned}
\tag{D.14}$$

The differential relations on these basis forms are derived from

$$\begin{aligned}
de^1 &= \alpha e^{72} - \frac{1}{2} e^{36} + \frac{1}{2} e^{45}, \\
de^2 &= \alpha e^{17} - \frac{1}{2} e^{35} - \frac{1}{2} e^{46}, \\
de^3 &= \beta e^{74} + \frac{1}{2} e^{25} + \frac{1}{2} e^{16}, \\
de^4 &= \beta e^{37} - \frac{1}{2} e^{15} + \frac{1}{2} e^{26}, \\
de^5 &= -\gamma e^{67} + \frac{1}{2} e^{14} - \frac{1}{2} e^{24},
\end{aligned}
\tag{D.15}$$

⁸The case where $p = q = 0$ is the trivial fibration case where the coset becomes $[SU(3)/U(1) \times U(1)] \times U(1)$. In that case this is the same as compactifying type IIA supergravity on the manifold $SU(3)/U(1) \times U(1)$.

$$de^6 = \gamma e^{57} - \frac{1}{2}e^{13} - \frac{1}{2}e^{24},$$

$$de^7 = -\alpha e^{12} - \beta e^{34} - \gamma e^{56}.$$

These then give the differential relations (5.14) where we have defined the structure constants

$$\alpha \equiv f^7_{12} = \frac{q}{\sqrt{3p^2 + q^2}},$$

$$\beta \equiv f^7_{34} = \frac{3p + q}{2\sqrt{3p^2 + q^2}},$$

$$\gamma \equiv f^7_{56} = \frac{3p - q}{2\sqrt{3p^2 + q^2}}.$$
(D.16)

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